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How to Deal with Partial Information?

The Case of P. Berlin 6619

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How to Deal with Partial Information?

The Case of P. Berlin 6619

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MATHEMATICAL PAPYRI, as other papyri, can be fragmentary. Even if the presence of numbers and operations of calculation can be useful in the reconstruction of texts and procedures, there is the risk of proposing fanciful interpretations, especially when the available information are scanty. The present article aims to propose some guidelines, in particular with reference to problem 1 of P. Berlin 6619, in which the final part of each line is lost. As I have indicated in a previous work, the most plausible hypothesis is that this problem deals with the calculation of a rectangle in "false position", with the aim of showing a case of the "Pythagorean theorem".[1]

The knowledge of the "Pythagorean theorem" in Egyptian mathematics is attested in four problems of the demotic papyrus Cairo JE 89127-30, 89137-43, from the Greco-Roman period. In three of these problems, the application of the "Pythagorean theorem" involves the triples 6-8-10, 10-10 ½-14 ½, and 5-12-13.[2] A term for the hypotenuse is not present there, as practical examples are applied to different positions of a "pole" (*ẖ.t*), which form a right-angled triangle.[3] In another problem solved with the "Pythagorean theorem", the word *ḳḥ* identifies the diagonal of a rectangle.[4]

Problem 1 of P. Berlin 6619 appears on the recto of the largest fragment of the papyrus, which is dated to the Middle Kingdom. A transcription of the text is shown in figure 1, followed by transliteration and translation.[5]

The interpretation I proposed for this problem text with the publication of the two main fragments of the papyrus can be summarized as follows:[6]

> The sides of *a rectangle* are calculated with the method of "false position", with the condition that they are in a ratio of 1 to ½ ¼, and that the sum of their squares is 100.

[1] L. MIATELLO, "A Debated but Little Examined Mathematical Text: Papyrus Berlin 6619", *ZÄS* 139, 2012, p. 158-170, Taf. 26–27.
[2] R.A. PARKER, *Demotic Mathematical Papyri*, Providence, 1972, p. 35-37.
[3] *Ibid.*, p. 35.
[4] *Ibid.*, p. 41-42.
[5] L. MIATELLO, *ZÄS* 139, Taf. 26 (bottom), p. 162-163, fig. 3.
[6] *Ibid.*, p. 162–169.

A different reading of the text and interpretation of the procedure was proposed at the beginning of last century by Schack-Schackenburg, and later followed by other scholars:[7]

> An area of 100 square cubits is distributed between *two squares*, whose sides, in a ratio of 1 to ½ ¼, are calculated with the method of “false position”.

In a recent article, the authors Gavin and Schärlig argue that my transcription and translation of the problem text is compatible with Schack-Schackenburg’s interpretation of the two squares, which they consider “the most plausible”.[8] Introducing themselves as mathematicians, they avoid to enter into the detail of the issues implied in the philological analysis of the text.

Evaluations of “compatibility” or “plausibility” should be based on the analysis of the text in the papyrus and on concrete experimentations of hypotheses of reconstruction, and not on abstract arguments. I will highlight here again the main issues of interpretation.

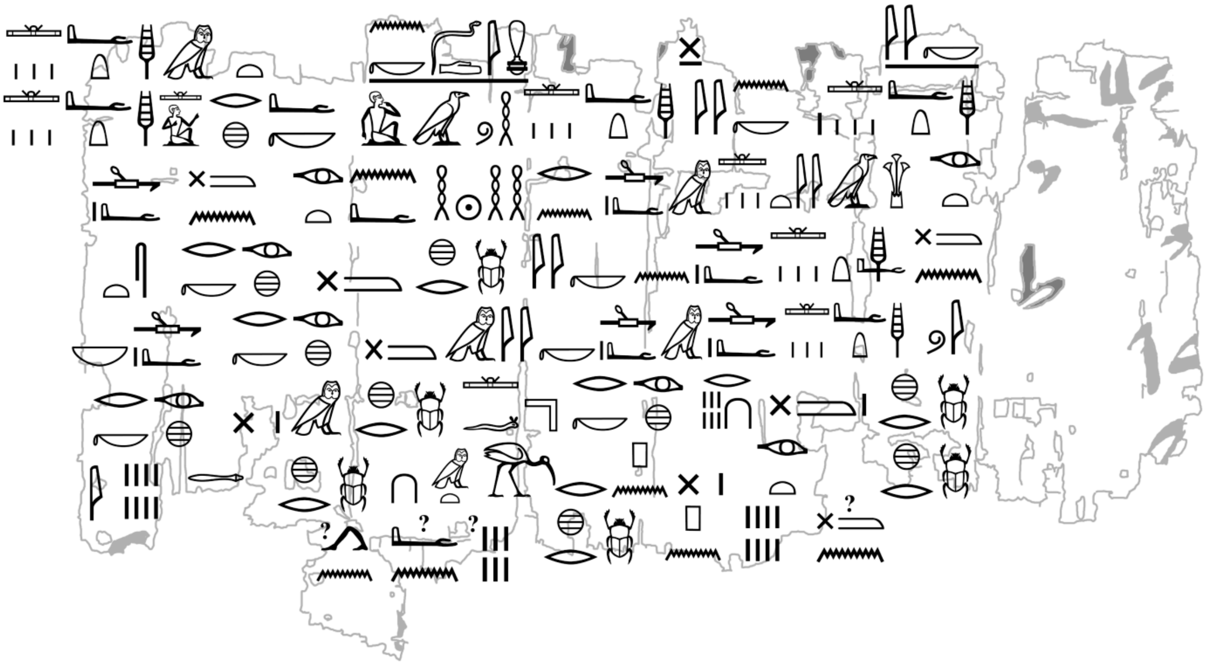

Fig. 1. Transcription of P. Berlin 6619, No. 1.

[1] *ky [... ½] ¼* (?) *[...] mj ḏd n=k [...]t m ꜥḥꜥ [...]* [2] *ꜥḥꜥ* 1 *n ky ꜥḥꜥ ḥꜣ dj=k rḫ=j ꜥḥꜥ[w...]* [3] *jr.t ḥꜣy.t m wꜥ r nḥḥ ḥnꜥ jr.t ½ ¼ n wꜥ [...]* [4] *½ ¼ n ꜥḥꜥ wꜥ n ky ḫpr.ḫr ½ ¼ jr.ḫr=k s.t [...]* [5] *jw ꜥḥꜥ wꜥ m wꜥ ky m ½ ¼ jr.ḫr=k wꜥ nb [...]* [6] *ḫpr.ḫr* 1 ½ ¼ (sic) $\frac{1}{16}$ *jr.ḫr=k ḳnb.t=f ḫpr.ḫr m* 1 ¼

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[7] H. SCHACK-SCHACKENBURG, “Der Berliner Papyrus 6619 (mit 1 Tafel)”, *ZÄS* 38, 1900, p. 135-140; S. COUCHOUD, *Mathématiques égyptiennes: recherches sur les connaissances mathématiques de l’Égypte pharaonique*, Paris, 1993, p. 131-136.

[8] J. GAVIN, A. SCHÄRLIG, “Fausse position et heuristique au Moyen Empire”, *ENiM* 8, 2015, p. 113-132, esp. 117-123.

jr.ḫr=k [...] 7 *ḫpr[.ḫr...] jr.t* 1 ¼ *pn r gm.t* 10 *ḫpr.ḫr ꜥꜣ[...]* 8 *j[...]* 8 *[... ½ ¼ n]* 8 *pn ḫpr.ḫ[r* 6 (?)*] ꜥn n (?) [...].*

1 <u>Another [… ½] ¼</u> (?) <u>[…] If one says to you:</u> […] as quantity […] 2 one quantity to the other quantity. Please let me know the quantity […]. 3 Calculation of the rectangle in 1 as constant, with the calculation of ½ ¼ of 1 […] 4 ½ ¼ of one quantity to the other. Then ½ ¼ results. Then you calculate it […]. 5 One quantity is 1, the other is ½ ¼, then you calculate every one […]. 6 Then 1 ½ <~~¼~~> 1/16 results. Then you calculate its square root. Then 1 ¼ results. Then you calculate […]. 7 [Then …] results. Division of 10 by this 1 ¼. Then [the proportion (?)] 8 results. […]. 8 [… ½ ¼ of] this 8. Then [6 (?)] results […].

The Application of the Method of "False Position" in the Problem Text

In line 3, the papyrus shows the following sentence:

3 *jr.t ḥꜣy.t m wꜥ r nḥḥ ḥnꜥ jr.t ½ ¼ n wꜥ [...].*

Calculation of the rectangle with 1 as constant, together with the calculation of ½ ¼ of 1 […].

A rectangle is calculated by assigning the "false" value 1 to one side and ½ ¼ to the other. At the end of line 3 we can reconstruct: [**for the other quantity/side**].

Assuming reconstruction at the end of line 3 of the sentence "together with the calculation of ½ ¼ of 1 [for the side of the *other* quadrilateral]", at the beginning we would expect to find: *jr.t ḥꜣy.t wꜥ* "calculation of *one* quadrilateral". Instead, there is only *ḥꜣy.t.*

The sentence *wꜥ r nḥḥ*, lit. "1 to the eternity" was incorrectly translated "1 all over", or "with 1 on all sides" by Schack-Schackenburg and Couchoud.[9] Instead, the scribe points out that the value 1 **for one quantity/side** is taken as "constant", or as value "(fixed) to the eternity" (*r nḥḥ*).[10]

In line 5, the "false position" is specified again: "**one quantity is 1, the other is ½ ¼**".

The term *ḥꜣy.t* is rarely attested. It is found also in P. Lahun UC32162-1, with reference to a rectangle (taken 10 times), and not a square.[11] The plural stokes give a sense of abstractness to the term, as in *ꜥḥꜥ* "quantity",[12] or *ꜣḥ.t* "field", "area".[13]

The Enunciation of the Problem

Let's consider now the enunciation of the problem, whose exact reconstruction is difficult, as the initial and final parts are lost. First, we observe that the value 100 is missing in the

[9] H. SCHACK-SCHACKENBURG, *ZÄS* 38, p. 137; S. COUCHOUD, *Mathématiques égyptiennes*, p. 132, n. 4.
[10] L. MIATELLO, *ZÄS* 139, p. 165.
[11] A. IMHAUSEN, *Ägyptische Algorithmen. Eine Untersuchung zu den mittelägyptischen mathematischen Aufgabentexten*, *AgAbh* 65, Wiesbaden, 2003, p. 79-80, 355.
[12] *Wb* I, 220 (15)-221 (1).
[13] E.g: P. Rhind, No. 49, lines 2 and 8; P.Rhind No. 50, lines 4 and 13; P. Moscow, No, 10, lines 4 and 13. Cf. A. IMHAUSEN, *Ägyptische Algorithmen*, p. 246-249, 322-323.

papyrus. It is implied in line 6, where the square root of 100 can be plausibly reconstructed, in consideration of the number 10 that appears in the next operation in line 7, and in parallel with the previous operation of square root yielding the value 1 ¼:[14]

jr.ḥr≈k [ḳnb.t n 100*]* 7 *ḫpr[.ḫr* 10*].*

Schack-Schackenburg proposed the following reconstruction of the enunciation of the problem in the first two lines:

<u>*(...) mj ḏd n≈k*</u> *[mḥ-tꜣ (?)...].t m ʿḥʿ [2 jr.t ½ ¼ n ... n]* 2 *ʿḥʿ wʿ n ky ʿḥʿ ḥꜣ dj≈k rḫ≈j ʿḥʿ [nb jm].*

<u>(…) If one says to you:</u> [100 square cubits (?) …] as [two] quantities, [and calculation of ½ ¼ of the side of] 2 one quantity to the other quantity. Please let me know [all] quantities [therein].[15]

Not only this rendering is very confused, but it is also fully untenable. A measure of 100 square cubits would be calculated (distributed?) as two *ʿḥʿ* quantities. No measure in cubits or other metrological quantity appears anywhere in the text, but this is the least of the inconveniences. Two *ʿḥʿ* "(unknown) quantities" would be the *areas of two squares*. The condition that the two sides of the squares are in a ratio of 1 to ½ ¼ would be given by the following sentence: "[½ ¼ of the side of] one quantity to the other quantity". This is untenable, as in the second line one does not read "to the *side* of the other quantity", but *n ky ʿḥʿ* "to the other quantity", which is assumed to be the second area. The confusion is such that Couchoud writes: "la relation de proportion liant les deux surfaces est de 1 a ½ ¼".[16]

As Schack-Schackenburg is convinced that two squares are implied, he introduces two *ʿḥʿ* quantities as areas. There would be *four ʿḥʿ* quantities: two areas and two sides, but only *two ʿḥʿ* quantities, corresponding to the sides, are indicated in the operations of calculation.[17] The sentence "½ ¼ of one quantity to the other" in line 4 is referred to two sides! Then he introduces artificially the further concept of "[side of] one quantity", which, as we have seen, is untenable. In any case, the confusion is huge, and the hypotheses are not feasible.

In Middle Egyptian mathematical papyri a number or measurement is never given right after *mj ḏd n≈k* in the calculation of geometric objects or other objects. When this expression appears after the title of the problem, the scribe usually mentions *immediately the object being calculated*, as shown in the following list.[18]

Moscow mathematical papyrus:

- *mj ḏd n≈k ḥmw*: "rudder" (No. 2).
- *mj ḏd n≈k ḫ.t-ṯꜣw* : "mast" (No. 3).

[14] Contrarily to what Gavin and Schärlig maintain (in *ENiM* 8, p. 121), this reconstruction of the number 100 is clearly indicated in my article: L. MIATELLO, *ZÄS* 139, p. 163-164.
[15] Cf. H. SCHACK-SCHACKENBURG, *ZÄS* 38, p. 136-137. The author writes "100 Quadratellen", without indicating the corresponding term *mḥ-tꜣ*. On measures of area: A. IMHAUSEN, *Ägyptische Algorithmen*, p. 65-68.
[16] S. COUCHOUD, *Mathématiques égyptiennes*, p. 133.
[17] Only one *ʿḥʿ* is determined in non-geometric *ʿḥʿ* problems: A. IMHAUSEN, *Ägyptische Algorithmen*, p. 36-53. For a brief description: L. MIATELLO, "The Difference 5½ in a Problem of Rations from the Rhind Mathematical Papyrus", *Historia Mathematica* 38, 2008, p. 279-280.
[18] Cf. A. IMHAUSEN, *Ägyptische Algorithmen*, p. 197-348.

- *mj ḏd n≠k spd.t*: "triangle" (No. 4, 7, 17).
- *mj ḏd n≠k t.w*: "bread loaves" (No. 5, 8, 20).
- *mj ḏd n≠k ʿ.t*: "(rectangular) room" (No. 6).
- *mj ḏd n≠k nb.t*: "basket" (No. 10).
- *mj ḏd n≠k* : "truncated pyramid" (No. 14).
- *mj ḏd n≠k ḥnḳ.t*: "beer" (No. 16).
- *mj ḏd n≠k r*: "part of garment" (No. 18).

Rhind mathematical papyrus:

- *mj ḏd n≠k jfd*: "rectangle" (No. 49).
- *mj ḏd n≠k spd.t*: "triangle" (No. 51).
- *mj ḏd n≠k qrf.t nbw*: "bag of gold" (No. 62).
- *mj ḏd n≠k t.w*: "bread loaves" (No. 72, 73, 78).
- *mj ḏd n≠k ḥnḳ.t*: "beer" (No. 77).

In my previous contribution I proposed that the word *jfd* () "rectangle" was written after *mj ḏd n≠k*.[19] The sentence *mj ḏd n≠k jfd* appears in P. Rhind, No. 49.[20] On the lower edge there is a small trace in black ink compatible with the extremity of an oblique sign, as that of the viper. A possible restoration of the hieratic text is shown in figure 3a.

The *t* sign presumably belongs to the word *jr.t* "calculation", also considering that on the leftmost edge there is a small trace in black ink [fig. 2b], whose shape is consistent with the restoration of the eye sign.[21] In any case, there is no space for *jfd* 2 "two quadrilaterals", which would have been written with two vertical strokes on the left of the determinative.[22] Also, while the calculation of a rectangle is defined by the two dimensions of the object, the sentence "two squares calculated as two quantities" would be ambiguous, as these quantities could be the areas of the squares. We should expect to find: "the *sides* of two squares are calculated as two quantities".

a **b**

Fig. 2. **a**: Hypothesis of restoration of the hieratic text in the first line; **b**: Detail of trace.

[19] L. MIATELLO, *ZÄS* 139, p. 164. Cf. *Wb* I, 71, 12.
[20] Cf. A. IMHAUSEN, *Ägyptische Algorithmen*, p. 247, line 1.
[21] L. MIATELLO, *ZÄS* 139, p. 164.
[22] S. Couchoud (in *Mathématiques égyptiennes*, p. 133) proposes: "Sont donnés deux rectangles dont la somme des surfaces est égale à 100 (…)", without proposing a restoration for the terms "two rectangles". Numerical adjectives were written in the Middle Kingdom with long vertical strokes.

At the end of line 2 the scribe presumably gives as condition that the sum of the squares of the two *ʿḥʿ* quantities is 100. This sum corresponds to the square of the diagonal of the rectangle that is being calculated, indicating the didactical aim of showing a case of the "Pythagorean theorem".[23]

> *(...) mj ḏd n≠k [jfd jr].t m ʿḥʿ [½ ¼ n]* $\overset{2}{|}$ *ʿḥʿ* 1 *n ky ʿḥʿ ḥꜣ dj≠k rḫ≠j ʿḥʿ [*2 *dmḏ m sn r* 100*]*.
>
> (…) If one says to you: [a rectangle, calculation] as quantity, [½ ¼ of] $\overset{2}{|}$ one quantity to the other quantity. Please let me know the [two] quantities, [adding up as square to 100].[24]

A Rectangle Calculated in "False Position" and the "Pythagorean Theorem"

As we have seen, the analysis of the text indicates that the hypothesis of the two squares is unsupported and unsuitable, while the hypothesis of the rectangle is straightforward. A rectangle is calculated in several problems in Middle Egyptian mathematical papyri, even by using the ratio of 1 to ½ ¼ between the sides.[25] This ratio appears also in the calculation of pyramids in the Rhind mathematical papyrus, as the *sḳd* "slope" of 5 palms and 1 finger of several of these pyramids corresponds to the base ½ ¼ cubits of a right-angled triangle with height 1 cubit.[26] This is the *sḳd* of the pyramids of Khafra in Giza, Userkaf, Neferhetepes, Pepi I and Pepi II in Saqqara.[27] Further architectural parallels are shown in my previous contribution,[28] in which I indicated also the similarity of the Egyptian algorithm with the calculation of a rectangle with ratio of 1 to ½ ¼ between the sides in an Old Babylonian clay tablet from Susa, dating to the first half of the second millennium B.C.[29]

The two problems are compared in Table 1, by means of the algorithmic method.[30] The numbers in the Babylonian text are in sexagesimals, and translations into decimals are shown between brackets.

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[23] L. MIATELLO, *ZÄS* 139, p. 166.

[24] For an example of the word *sn* "square (of a number)", with the legs-walking sign (D54), see P. Moscow, No. 14: A. IMHAUSEN, *Ägyptische Algorithmen*, p. 330-331 (lines 4 and 6, and illustration).

[25] This ratio between the sides of a rectangle appears in P. Moscow, No. 6, and P. UC32162-1: A. IMHAUSEN, *Ägyptische Algorithmen*, p. 78-80, 314-315, 355. See also: L. MIATELLO, "Transformations of Geometrical Objects in Middle Egyptian Mathematical Texts", *Physis* 48, 2011-2012), p. 6-9.

[26] P. Rhind, No. 57, 58, 59, and 59b. Cf. A. IMHAUSEN, *Ägyptische Algorithmen*, p. 260-265.

[27] Cf. C. ROSSI, *Architecture and Mathematics in Ancient Egypt*, Cambridge, 2004, p. 245-250.

[28] L. MIATELLO, *ZÄS* 139, p. 166-167.

[29] J. HØYRUP, *Lengths, Widths, Surfaces. A Portrait of Old Babylonia Algebra and its Kin*, New York, 2002, p. 194-195. The tablet is published in: E.M. BRUINS, M. RUTTEN, *Textes mathématiques de Suse*, *Mission archéologique en Iran* 34, Paris, 1961, p. 101-102, pl. 12. See also the translation with the algorithm in L. MIATELLO, *ZÄS* 139, p. 167-168.

[30] On this method: J. RITTER, "Chacun sa vérité: les mathématiques en Égypte et en Mésopotamie", in M. Serres (ed.), *Éléments d'histoire des sciences*, Paris, 1989, p. 39-61; A. IMHAUSEN, *Ägyptische Algorithmen*, p. 25-26.

		P. Berlin 6619, 1			***Babylonian TMS 19, 1***
	D_1	[$\frac{1}{2}\ \frac{1}{4}$] (ratio 1 : $\frac{1}{2}\ \frac{1}{4}$)		D_1	0;15 (= $\frac{1}{4}$) (→ ratio 1 : $\frac{1}{2}\ \frac{1}{4}$)
	D_2	[100] (→ diagonal^2)		D_2	0;40 (= $\frac{2}{3}$) (diagonal)
(1)	$1 \cdot D_1$	$1 \cdot \frac{1}{2}\ \frac{1}{4} = \frac{1}{2}\ \frac{1}{4}$	(1)	$1 - D_1$	1 – 0;15 = 0;45 (= $\frac{1}{2}\ \frac{1}{4}$)
(2)	1^2	[$1^2 = 1$]	(2)	1^2	$1^2 = 1$
(3)	$(1)^2$	[$\frac{1}{2}\ \frac{1}{4}{}^2 = \frac{1}{2}\ \frac{1}{16}$]	(3)	$(1)^2$	$0;45^2$ = 0;33.45 (= $\frac{1}{2}\ \frac{1}{16}$)
(4)	(2) + (3)	[1 + $\frac{1}{2}\ \frac{1}{16}$] = 1 $\frac{1}{2}\ \frac{1}{16}$	(4)	(2) + (3)	1 + 0;33.45 = 1;33.45 (= 1 $\frac{1}{2}\ \frac{1}{16}$)
(5)	$\sqrt{(4)}$	$\sqrt{1\frac{1}{2}\frac{1}{16}} = 1\ \frac{1}{4}$	(5)	$\sqrt{(4)}$	$\sqrt{1;33.45}$ = 1;15 (= 1 $\frac{1}{4}$)
(6)	$\sqrt{D_2}$	[$\sqrt{100} = 10$]			
			(6)	1 : (5)	1 : 1;15 = 0;48 (= 1 : 1 $\frac{1}{4}$)
(7)	(6) : (5)	10 : 1 $\frac{1}{4}$ = [8]	(7)	$D_2 \cdot (6)$	0;40 · 0;48 = 0;32
			(8)	1 · (7)	1 · 0;32 = 0;32
(8)	(1) · (7)	[$\frac{1}{2}\ \frac{1}{4}$] · 8 [= 6]	(9)	(1) · (7)	0;45 · 0;32 = 0;24

Tab. 1. Comparison of the algorithms in P. Berlin 6619, No. 1, and in a Babylonian tablet.

As in the Babylonian tablet, problem 1 shows the following operations, executed step by step:

- Square of 1.
- Square of ½ ¼.
- Sum of the squares of 1 and ½ ¼.
- Square root of the sum of the squares (= 1 ¼).

The diagonal of the "false" rectangle (= 1 ¼) is calculated as the square root of the sum of the squares of the "false" *ʿḥʿ* quantities. As the square of the diagonal is equal to the sum of the squares, the square root of the given value 100 is the true diagonal. Then the division of 10 by 1 ¼ gives the proportionality factor 8, which, multiplied by 1 and ½ ¼, yields the true quantities 8 and 6.[31] A geometric representation of the procedure is proposed in figure 3.[32]

[31] The multiplication of the proportionality factor by 1 is omitted in the papyrus, as in most non-geometrical *ʿḥʿ* problems: cf. A. IMHAUSEN, *Ägyptische Algorithmen*, p. 41, 52.

[32] L. MIATELLO, *ZÄS* 139, p. 167, fig. 4. In a recent work on Egyptian mathematics, the implication that the sum of the squares of the quantities is given as datum, and not the diagonal as in the Babylonian problem, is considered a negative facet of my interpretation: M. MICHEL, *Les mathématiques de l'Égypte ancienne*, Bruxelles, 2014, p. 225-226. This is a subjective argument, as the sum of the squares of the "false" quantities appears also in the Babylonian algorithm.

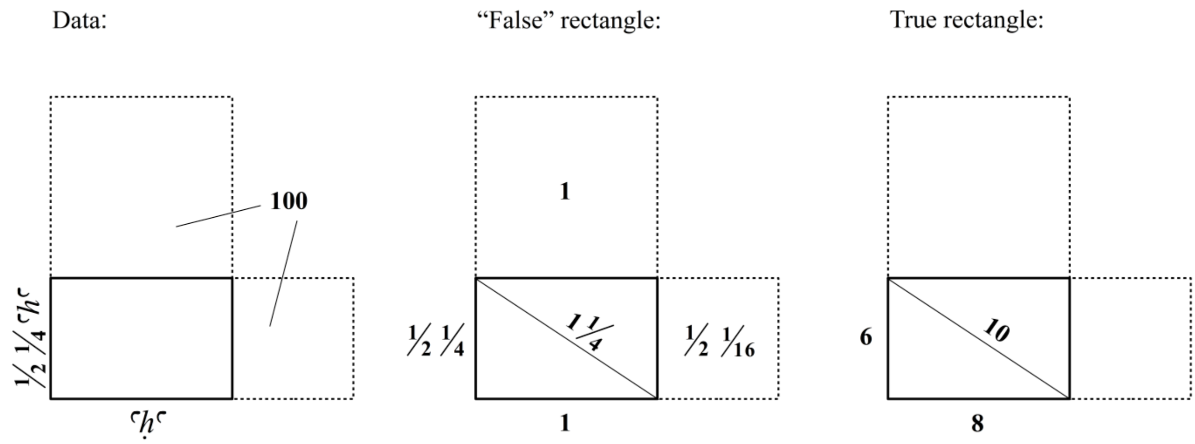

Fig. 3. Geometric representation of the algorithm in P. Berlin 6619, No. 1.

This procedure implies two ꜥḥꜥ quantities, corresponding to the sides, as indicated in the papyrus. Schack-Schackenburg’s hypothesis of four ꜥḥꜥ is redundant and incorrect also from a mathematical point of view. In fact, only the sides are ꜥḥꜥ; the areas are not called ꜥḥꜥ, as they are the squares of the values 1 and ½ ¼ determined with the method of “false position”.

In order to construct the casing stones of the pyramids the masons probably used a wooden triangle,[33] which in the pyramid of Khafra and several pyramids of the fifth and sixth dynasty would have had base ½ ¼ cubits and height 1 cubit (*sḳd* of 5 palms, 1 finger). The hypotenuse 1 ¼ of the triangle could be easily ascertained empirically, and problem 1 of P. Berlin 6619 indicates that in the Middle Kingdom the Egyptians calculated mathematically the diagonal 1 ¼ of a rectangle with ratio of 1 to ½ ¼ between the sides.

I will now comment on another problem in the papyrus, which, according to some scholars, would concern the distribution of the area of a square between two squares.

Problem 3 of P. Berlin 6619

Problem 3 appears on the recto of a small and damaged fragment of the papyrus. A transcription of the problem is shown in figure 4, followed by transliteration and translation.[34]

[33] See M. LEHNER, *The Complete Pyramids*, London, 1997, p. 220.
[34] L. MIATELLO, *ZÄS* 139, Taf. 26 (top), p. 169-170, fig. 5.

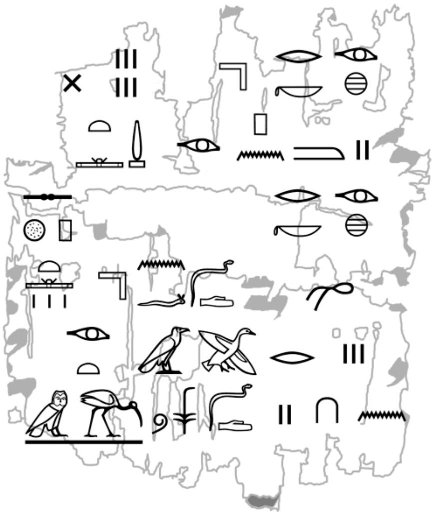

Fig. 4. Transcription of P. Berlin 6619, No. 3.

$\overset{x+1}{|}$ *[...] jr.ḫr≠k qnb.t [...]* 6 ¼ *[...]* $\overset{x+2}{|}$ *[...]* 2 ½ *pn jr ḏꜣ.t [...]* $\overset{x+3}{|}$ *[...] jr.ḫ[r≠k...] sp [...]* $\overset{x+4}{|}$ *[...] ḏd.n≠f ḳnb.t [...]* $\overset{x+5}{|}$ *[...t]* 3 *r pꜣ jr.t [...]* $\overset{x+6}{|}$ *[...] n* 12 *ḏd sw <u>gm[j≠k nfr</u> (?)].*

$\overset{x+1}{|}$ […] Then you calculate the square root […] 6 ¼ [...] $\overset{x+2}{|}$ […] this 2 ½, calculating (?) the rest […] $\overset{x+3}{|}$ Then [you] multiply […] by […] $\overset{x+4}{|}$ […] He has said: the square root […] $\overset{x+5}{|}$ […] 3 to this, calculation (?) […] $\overset{x+6}{|}$ […] of 12 says it, [<u>what you have found is correct</u> (?)].

As indicated also by Imhausen, the information are insufficient to reconstruct the procedure.[35] Only four numbers can be read, and the rigorous reconstruction of the data and the enunciation of the problem is impossible. Even the object of calculation is unknown.

Schack-Schackenburg proposed an interpretation of problem 3 as variant of problem 1, with 400 instead of 100;[36] another variant based on the hypothesis of the two squares has been proposed recently by Gavin and Schärlig, with 225 instead of 100.[37] Both interpretations are not supported by evidence. Nothing suggests that 6 ¼ is the result of the square of 2 and 1 ½. It could be a datum, or the result of other operations. In the following hypothetical example, 6 ¼ is the result of an operation of multiplication:

A *šfd.w* "sheet of papyrus" () of side 6 and 10 is subdivided into 12 rectangular sections with a ratio of 1 to 1 ¼ between the sides. The area of each rectangle is 5, which, multiplied by 1 ¼, gives 6 ¼. The sides of the rectangles are equal to the square root of 6 ¼, i.e. 2 ½. Calculating the rest of the sides: 5 divided by 2 ½ is 2. It's been said that the square

[35] A. IMHAUSEN, *Ägyptische Algorithmen*, p. 175, 361; L. MIATELLO, *ZÄS* 139, p. 169-170.
[36] H. SCHACK-SCHACKENBURG, "Das kleinere Fragment des Berliner Papyrus 6619", *ZÄS* 40, 1902, p. 65-66.
[37] J. GAVIN, A. SCHÄRLIG, "Fausse position et heuristique au Moyen Empire", *ENiM* 8, 2015, p. 123-126.

root of 6 ¼ gives the width 2 ½ of each rectangle, and their height is 2. The sheet has an array of 3 rectangles by 4 rectangles, for a total of 12.[38]

This procedure reconciles with all numbers and operations in the small fragment and it is even similar to that in the fragment UC32162-1 from Lahun, but this does not mean that the reconstruction is correct. This example simply demonstrates that the available information are insufficient and it is not possible to reconstruct the procedure with mathematical suppositions only.

In any case, problem 3 is presumably *substantially different* from problem 1, as indicated also by the term *ḏꜣ.t* and the band-of-string sign (V12). Middle Kingdom Mathematical papyri were several meters in length, and this problem could deal with anything.

It is interesting to note, however, that in Middle Egyptian mathematical texts operations of square root (*ḳnb.t*) allow to determine either the side of a rectangle (P. Moscow, No. 6 and P. UC32162-1), or the length of a triangle calculated as side of a rectangle (P. Moscow, No. 7 and 17).[39] Mathematical problems usually deal with the calculation of rectangular, non-square rooms and fields, as the typical shape of rooms in houses, tombs, and temples, and that of the cultivated fields, was rectangular. The only example of a square field is P. Rhind, No. 48, in which the areas of a square and an inscribed circle are calculated.[40]

Conclusions

I have considered interpretation issues of problem 1 in P. Berlin 6619. In conditions of partial information, a procedure should not be reconstructed only on the basis of mathematical reasonings. In the case of Problem 1, the hypothesis of reconstruction of the procedure cannot prescind from concrete hypotheses of reconstruction and restoration of the text. The analysis of the text suggests that this problem deals with the calculation of the sides of a *rectangle*. Previous hypotheses on the calculation of two squares are unsupported and unsuitable. The value 1 ¼ in the algorithm of the problem is presumably the diagonal of a rectangle with a ratio of 1 to ½ ¼ between the sides, which are calculated with the method of “false position” with the aim of showing a case of the “Pythagorean theorem”, as in an Old Babylonian clay tablet.[41] This knowledge should by no means surprise, considering that the Moscow mathematical papyrus shows an algorithm to determine the volume of a truncated pyramid, whose calculation in European mathematics does not occur before Fibonacci (thirteenth century).[42] Not necessarily, however, the knowledge of specific cases of the “Pythagorean theorem” implies its general applicability: the lack of a method to calculate the square root of any number would have limited the generalization of the algorithm.

[38] Grids were commonly used in administration documents to host data for accounts, lists, inventories and rosters, and in mathematical papyri.

[39] A. IMHAUSEN, *Ägyptische Algorithmen*, p. 314-316; 334-335; 355.

[40] *Ibid.*, p. 72-73, 244.

[41] J. Gavin, A. Schärlig (*ENiM* 8, 2015, p. 121-122) disregard completely the striking parallel, excluding with exclamation marks even the possibility that the Egyptians knew a case of the “Pythagorean theorem” as the Babylonians. Criticisms are expressed *ibidem* to the “excess of algebra” by other authors and a “heuristic” method is proposed, but there is no mention of the algorithmic method by Ritter and Imhausen (see note 30), which is the best alternative to algebraic renderings.

[42] L. MIATELLO, *Physis* 48, p. 19-23.

Résumé :

Dans le problème 1 du P. Berlin 6619, la dernière partie de chaque ligne est perdue. Dans cet article, j'indique les raisons pour lesquelles l'interprétation de la manière de procéder ne peut se fonder uniquement sur des raisonnements mathématiques, mais aussi sur l'expérimentation concrète d'hypothèses de reconstruction des phrases manquantes. L'analyse du texte du problème indique que les côtés d'un rectangle sont calculés avec l'objectif de montrer un cas du « théorème de Pythagore ». Une autre hypothèse, avancée au début du siècle dernier, est infirmée.

Abstract:

In problem 1 of P. Berlin 6619 the final part of each line is lost. In this paper I indicate the reasons why the interpretation of the procedure cannot be based only on mathematical arguments, but also on concrete experimentations of hypotheses of reconstruction of the sentences and restoration of the missing parts. The analysis of the problem text indicates that the sides of a rectangle are calculated with the aim of showing a case of the "Pythagorean theorem". An alternative hypothesis, advanced at the beginning of last century, is unsupported and unsuitable.

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