Building pyramids

Reconstructing the process of lifting stones

(1) Bernard Mullers, (2) Andrew Borgart, (3) Olaf E. Kaper, Bert Bast, (4) Egbert Kuipers

(1) mullers.bernard@gmail.com, (2) Delft University of Technology, Dept Structural Mechanics, (3) Leiden University Institute for Area Studies, chair of Egyptology, (4) Private Kromhout Museum of Historical Engines, Zwartsluis.

THE PROCESS of building the pyramids leaves us with many unsolved questions as yet. Many aspects about the social infrastructure,[1] source of their limestone,[2] used tools[3] and transportation[4] have been described, but it is not yet clear and surely not undisputed, in what way the heavy building stones found their right way and position in the ever higher rising building. The largest pyramids have been built during the 4th Dynasty in the bronze age, between c. 2543 and 2442 B.C. All were built within this relatively short time span of 100 years. Apart from their function as funerary monuments, they may be seen metaphorically as the building blocks of the early Egyptian state.[5] The largest pyramid of all, the Great Pyramid of Cheops has a ground plan of 230 metres by 230 metres. Its original height was 146 metres,[6] with at that time a special stone on top, the pyramidion. The commonest used building stones, lower in the pyramid, weighed approximately 2500 kg and most measured 1.27 m in width, 0.8 m in depth and 1.27 m in height.[7] The run of the steps in the lower part[8] of the pyramid is calculated with the rise 1.27 m, as approximately 1 m [fig 1]. The Great Pyramid comprised about 2.3 million building stones. It has been estimated that 11.000 up to 28.000 people were required to build the pyramid within 20 years and that each 4 minutes such a heavy stone had been placed.[9] That would require an extensive organization of labour with yet a simple and flexible infrastructure. Whereas the principle of tilt levering

---

[1] M. LEHNER, Z. HAWASS, *Giza and the pyramids: the definitive history*, London, 2017, p. 404.

[2] D. KLEMM, R. KLEMM, *The stones of the pyramids: provenance of the building stones of the Old Kingdom pyramids of Egypt,* Berlin, New York, 2010, p. 85-89; and M. LEHNER, Z. HAWASS , *op. cit.*, p. 424.

[3] M. VERNER, *The pyramids: the archaeology and history of Egypt's iconic monuments*, new and updated edition, Cairo, 2020, p. 409-410.

[4] F. MONNIER, *L'univers fascinant des pyramids d'Égypte*, Dijon, 2021, p. 223-231.

[5] P. TALLET, M. LEHNER, *The Red Sea scrolls: how ancient papyri reveal the secrets of the pyramids*, London, 2021, p. 284-302.

[6] Figures according to M. LEHNER, Z. HAWASS, *op. cit.*, p. 143.

[7] M. VERNER, *op. cit.*, p. 403, and M. LEHNER, Z. HAWASS, *op. cit.*, p. 142-43 point out that the core of the pyramid also contained many irregularly shaped blocks.

[8] In the upper part of the pyramid smaller building blocks have been used, not seriously implicating the method described here.

[9] H.J. DE HAAN, *Large Egyptian pyramids: modelling a complex engineering project*, Oxford, 2010, p. 50; F. MÜLLER-RÖMER, *Der Bau der Pyramiden im Alten Ägypten*, Munich, 2011, p. 406 arrives at an average of 435 stones blocks per day, even more than the 360 calculated by De Haan.

the stones, as introduced below, would work for all Old Kingdom pyramids, the calculations were made using the data of the Great Pyramid of Cheops.

Cheops:

$l = 230\,m$

$h = 146\,m$

$\textit{height of block} = \textit{rise of step} = 1.27\,m$

$\bar{x} = \textit{run of step}$

$$tg\beta = \frac{146}{230/2} = \frac{1.27}{\bar{x}} \Rightarrow \bar{x} \approx 1.00\,m$$

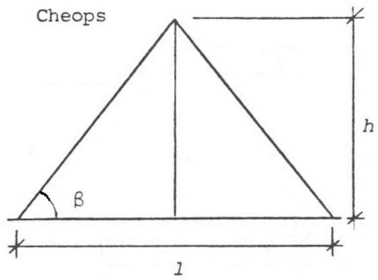


Fig. 1. Calculation of the run, using the height of a stone layer and the slope in degrees.

The stones of the pyramid of Cheops were mostly derived from limestone quarries close-by,[10] only the white limestone for the lining of inner spaces and for the outer casing came from the other, the eastern side of the river Nile from the quarries at Tura. The granite used for building the burial chamber and the relieving chambers above it came from Aswan.[11] Wheels as a tool, or pullies had not been invented at that time.[12] However, copper utensils such as chisels and drills[13] enabled the manufacture of pegged wood connections as notches, mortice and tenon. Palm fibre or papyrus rope enabled not only pulling on an object, but also to make lashings. For horizontal transport the stones were often placed on wooden sledges.[14] To cross the river Nile for transporting limestone or granite, boats were used,[15] see fig 2.

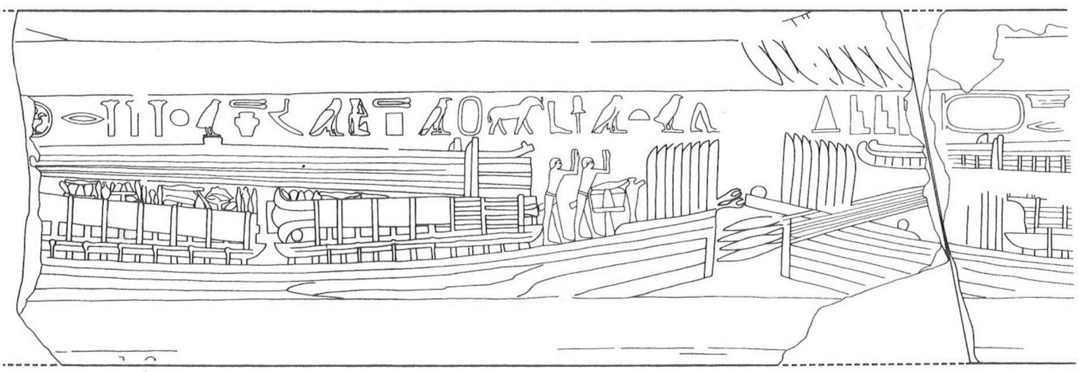


Fig. 2. Cargo boat loaded with two granite columns fixed on wooden sledges.[16]

[10] D. Klemm, R. Klemm, *op. cit.*, p. 85-89; M. Lehner, Z. Hawass, *op. cit.*, p. 424.
[11] *Ibid.*, p. 404.
[12] M. Isler, *Sticks. Stones, and shadows: building the Egyptian pyramids*, Oklahoma, 2001, p. 262.
[13] *Ibid.*, p. 229.
[14] D. Arnold, *Building in Egypt: pharaonic stone masonry*, New York, 1991, p. 276; M. Lehner, Z. Hawass, *op. cit.*, p. 414.
[15] M. Lehner, Z. Hawass, *op. cit.*, p. 410-411.
[16] A. Labrousse, A. Moussa, *La chaussée du complexe funéraire du roi Ounas*, Cairo, 2002, p. 141.

The wooden sledges were made to slide over mud, rounded desert sand, or they were transported horizontally using roller poles; these movements were all based on manpower. All was available, though wood such as cedar was precious, as it had to be imported over sea from the Levant. Cedar wood could be selected and cut in the Levant according to need. The growth of a cedar tree depends on its environment. In a dense forest they grow in a straight trunk towards the light, while free-standing cedars branches widely. Cedar wood might have been transported from there on board or alongside sail- and/or rowing boats, but it is more likely for easy transport of bigger numbers that they were towed for transport to Egypt, tied together and stacked up into large rafts.

This paper deals with the problem of vertical transport. There are no depictions or descriptions of the methods employed for building the Old Kingdom pyramids, and currently the most widely held theory assumes that ramps were built of mudbrick or stone rubble in order to drag the stones upward. Many different scenarios for the orientation of such ramps have been devised,[17] such as tangential and spiral ramps, but there remain great problems with this theory, mainly in the additional labour such ramps require. Recently, Tallet and Lehner have estimated that the total volume required for ramps around the building site and leading up to the top would be "86 per cent of the volume of the Great Pyramid".[18] All that mass of material needed to be removed after the work on the pyramid was finished and there is no archaeological evidence for such massive dumps at Giza or at other sites. Others have supposed massive scaffolding with bricks,[19] or lifting techniques and devices such tugging,[20] lever operations with a pole,[21] supporting slideways,[22] pulling combined with levers,[23] special frameworks,[24] counterweighted transport,[25] opposite stairway transport,[26] fulcrums,[27] and tumbling.[28] They all seem very unlikely[29] or even impossible to us,[30] and all give for sure "trouble at the top".[31] That is to say, whichever lifting methods are suggested for placing the pyramidion at the top, they all have not dealt with that problem properly.

---

[17] M. ISLER, *op. cit*,, p. 211-216; F. MÜLLER-RÖMER, *op. cit.*, p. 261-301; F. MONNIER, *op. cit.*, p. 236-237.
[18] P. TALLET, M. LEHNER, *op. cit.*, p. 273.
[19] D, ARNOLD, *op. cit.*, p. 98-100.
[20] M. ISLER, *op. cit.*, p. 259-260.
[21] M. LEHNER, *The Complete Pyramids*, London, 1997, p. 209.
[22] M. ISLER, *op. cit.*, p. 260
[23] *Ibid.*, p. 261.
[24] F. CORRARD, *Le dipode: des mâts des voiliers de l'Ancien Empire à l'outil de levage et de minutention des blocs de pierre. BSFE* 140, 1997, p. 46-47; H.J. DE HAAN, *op. cit.*, p. 51-52.
[25] M. ISLER, *op. cit.*, p. 264.
[26] *Ibid.*, p. 264.
[27] *Ibid.*, p. 261.
[28] *Ibid.*, p. 256-264.
[29] H.J. DE HAAN, *op. cit.*, p. 78-79; a critical overview of 16 such instruments is provided in F. MÜLLER-RÖMER, *op. cit.*, p. 302-349.
[30] This applies explicitly to the theories in P. GARDE-HANSEN, *On the Building of the Cheops Pyramid*, Copenhagen, 1975, p. 30; D. ARNOLD, *op. cit.*, p. 98; F. CORRARD, *op. cit.*, p. 46-47; P. TALLET, M. LEHNER, *op. cit.*, p. 273-276; and P. TALLET, *Les papyrus de la mer Rouge* I. *Le « journal de Merer » (Papyrus Jarf A et B)*, *MIFAO* 136, Cairo, 2017, p. 29.
[31] Term from M. Lehner and Z. Hawass, (*op. cit.*, p. 418). Similarly, in M. LEHNER, *op. cit.*, p. 222-223, and H.J. DE HAAN, *op. cit.*, p. 50.

A new method

Our hypothetical new building method is based on the principle of a truss lever, a sturdy construction fixed to a loading floor. This lever works as an aid for stepwise tilting of the floor. The lever and floor form a truss construction that resembles a cage. This cage can be loaded. By tilting to one side and propping the other, alternatively in opposite directions the cage, as a whole, can be lifted. The space needed to use such a tilt levering cage is achieved by creating temporary niches on certain steps in line above each other in the pyramid. These successive niches form a giant stairway for the use of the tilt levering cage.

Principle of tilting

Here, we describe a construction and a relatively easy working method, based on techniques available in the 4th Dynasty, to enable specifically the vertical transport of heavy stones. The usage of levers has been proposed by others, but the huge forces required in such a method are difficult to obtain with wooden poles, due to the high internal bending forces and stresses in a single pole as a result of the cantilever principle. To obtain higher forces, a big lever may be constructed. In our successful, but unpublished experiments, two of those methods involved undesired, but necessary adjustments for every stone and risks of damaging these. In this article, we propose a method obviating these disadvantages, as well as industrializing the process of vertical transport.

Our novel idea utilizes the principle of limited tilting with a constructed truss lever above the loading floor. The idea of tilting an object must have been familiar in antiquity. The process is employed, for example, in keeling over a ship [32] by pulling the mast down to one side for the purpose of the maintenance of the bottom.

The method provides a theory indicating how human activity may have transported the building stones for a pyramid to great heights. It is based on the technical preconditions of that time, and we do have some supporting argumentation for this theory. The present paper aims to outline the method (as tested in full size and at scale 1:10), after which further evidence may be sought in texts and archaeological remains to substantiate the method at the actual building sites. The method is supported by the description of Herodotus, who described the building process of the Great Pyramid in the following terms:[33]

> "This pyramid was built like a flight of stairs, which some call battlements,[34] others, other platforms. [2] When they had completed the foundation level, they lifted the rest of the stones by means of a device made of short pieces of wood, raising them from the ground up to the first tier of stairs. [3] When a stone had been lifted to the first tier, it was mounted onto another lifting mechanism standing on that tier, and from there it was hoisted to the second tier and to yet another machine. [4] Either they employed as many devices as there were tiers, or perhaps they used one single machine that was easy to move from one tier to the next after they had removed the stone. After the structure was complete, they finished off the top tiers first and worked downward so that the lowest tiers at ground level were finished last. I give both

[32] H.J. DE HAAN, *op. cit.*, p. 52.

[33] HERODOTUS 2.125; translation A.L. Purvis, in R.B. STRASSLER, *The Landmark Herodotus*, *The Histories*, New York, 2007, p. 174.

[34] In the Greek text: κροσσαί. That word has been later explained by Aristarchos (according to the English translation, see https://logeion.uchicago.edu/κρόσσαι) as scaling ladders; a translation fitting even better our idea.

> explanations here exactly as they were presented to me.” [5] After the structure was complete, they finished off the top tiers first and worked downwards so that the lowest tiers at the ground level were finished last”.

Even though Herodotus wrote his description in the fifth century BCE, his words accurately describe the process outlined below.

The tilt levering cage

Our novelty is a tilt levering cage: a construction of a wooden loading floor fixed with a wooden truss high above it as a lever. Straight cedar logs were used for all parts of this tilt levering cage. Straight cedar trunks could be selected in the forests of the Levant according to need, selected by diameter and length, which limited extensive woodworking. The tilt levering cage consists firstly of a cage floor constructed of two parallel girders with a length about 1.5 metre and a distance between the cores about 1.5 metre. These girders are connected by pegged mortice and tenon to each other at the top by two sturdy beams. More towards both ends of the two girders are supporting four obliquely standing poles. These are about 7 metres long and connected to the girders by pegged mortice and tenon. The four poles are firmly connected to each other at the top using notched wood joints with a single inlaid crossbar, everything tightly tied with lashings. A high pointed pyramid-shaped construction is thus obtained, which functions as a rigid truss lever.

We have made such a working tilt levering cage of pine in real size [fig. 3-4] as well as at scale 1:10 also made of wood [fig. 5]. The wood connections are made by means of notched, pegged mortice and tenon constructions, combined on top with a single inlaid cross bar and lashings.

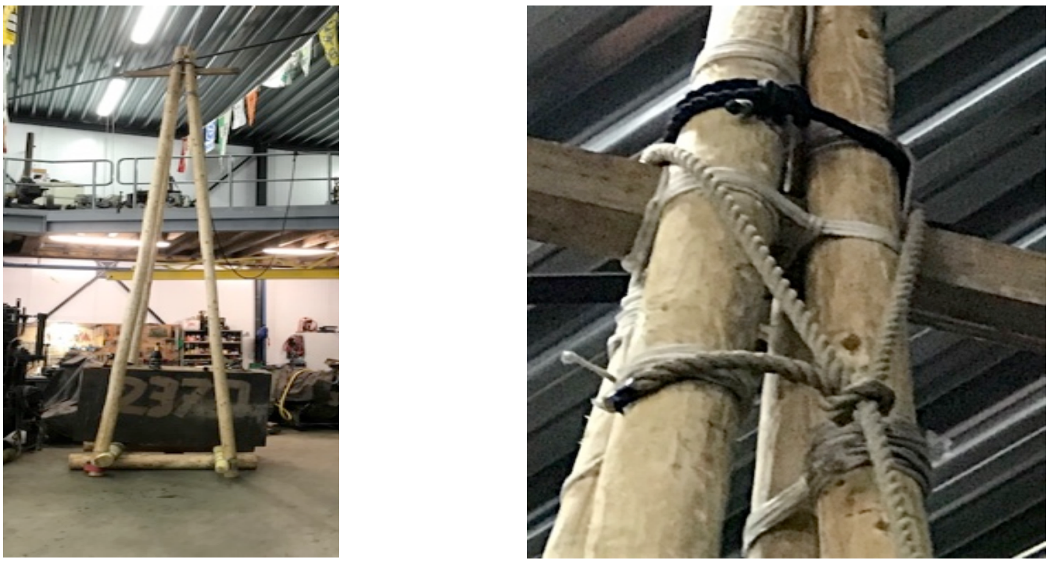


Fig. 3-4. A working tilt levering cage in reality (left) and a detail of the inserted cross bar at the top (right).

Fig. 5. A working model of a loaded tilt levering cage, Scale 1:10.

The construction of the tilt levering cage has some visual resemblance to the slightly tapered bipod mast of ships from the Old Kingdom period, which are nearly always two-legged.[35] This shows that there was ample experience with the selection of cedar wood, the construction, the wood connections and with the handling of such large wooden structures.

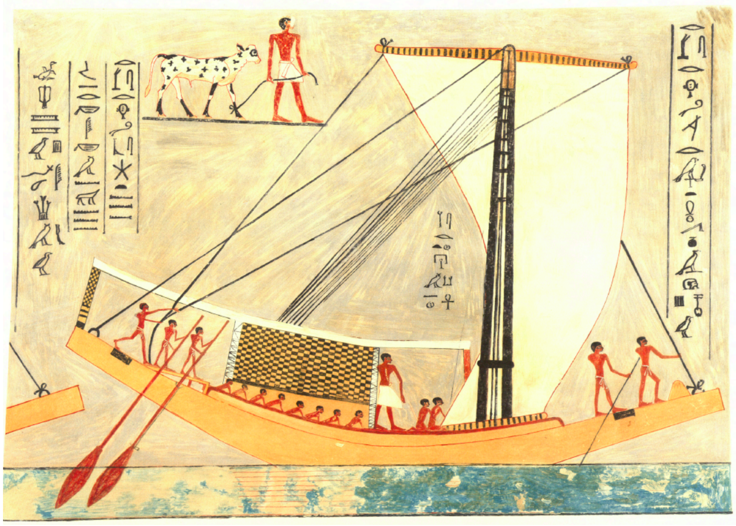


Fig 6. Boat with a bipod mast in a painting in the 5th Dynasty tomb of Kaiemankh (G4561) at Giza.[36]

[35] B. LANDSTROM, *Ships of the Pharaohs: 4000 years of Egyptian shipbuilding*, London, 1970, p. 35-69; D. JONES, *Boats*, London, 1995, p 37

[36] Facsimile painting from N.M. DAVIES, *Ancient Egyptian paintings: selected, copied, and described* I, Chicago, 1936, pl. 2. The two legs of this bipod mast are set crosswise to the length of the boat, thus giving more

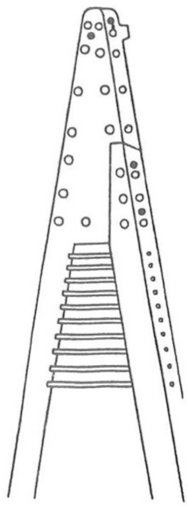


Fig. 7. Top of a bipod wooden mast with a rigid pegged wood construction.[37]

In the tilt levering cage presented here, the heavy object to be raised was moved horizontally onto the loading floor in the tilt levering cage. Thereafter the tilt levering cage was tilted over a girder, alternately to one side and the opposing one, using ropes attached to the top. After each lift due to the tilting, the upward displacement in height was secured by box cribbing. Two poles are placed on the floor and then a layer of two identical poles is added that are set perpendicular to the first two and so on. Thus, forming two stable box cribs as square hollow pillars of poles, under each girder, see fig. 8. A box crib pole is subjected to pressure in two places with both two contact points. For straight and stable box cribbing all poles had the same thickness between the two corresponding contact points.

The upwards straight cedar poles of the tilt levering cage have a length l [m] of about 7 metres and the horizontal pulling force needed for tilting is applied to the top of the form-stable tilt levering cage.

The great length of the poles and the form-stable tilt levering cage enables tilting quickly and reliably over and over again, thus to raise the tilt levering cage step by step. The girders serve not only to support the levering cage floor on which the object to be raised has been placed, but also as pivoting elements allowing the structure to be tilted, as well as elements by which, for example by using roller poles, the entire levering cage and its load can be moved horizontally.

---

rigidity where side stays are absent. An even stronger rigidity is obtained by the four legs of the tilt levering cage and its truss construction.

[37] D. JONES, *op. cit.*, p. 37. N.B. This pegged wood construction in the top should not be confused with the multiple transverse connections which are probably glide holes for lines for hoisting and lowering the square sail.

Fig 8. Tilting the model of the tilt levering cage and propping it through box cribbing forming two square stable hollow pillars.

The geometric principle of tilting is presented in fig. 9. The tilt levering cage rotates over the dashed red line, over the girders, when a horizontal force *H* [kg] is exerted at the top of the cage and perpendicular to the dashed red line. The opposite side displaces upward by *u* [m] and the created space can be fixed in the position by filling it up with planks. Each time the cage is tilted could raise the structure upwards with about *u* = 0.3 [m] as exemplified later. The distance measured from the bottom when the tilt levering cage is level to the centre of gravity of a stone *b* [m] is half the height of a stone plus the diameter of the girders.

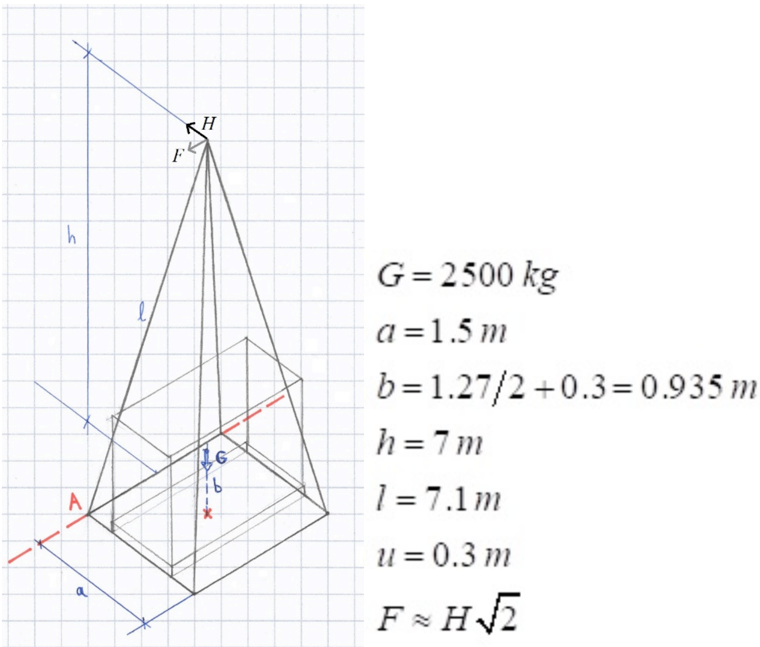


Fig. 9. Tilt levering cage, its geometry and measurements.

The relationship between the horizontal force *H* [kg] and the upward vertical displacement *u* [m] needs to be determined, the tilting formula, in order to assess whether it is possible to move the tilt levering cage upwards by human tugging, here e.g at an angle of 45 ° downwards, force *F* [kg]. This relation is non-linear because during the tilting the centre of gravity of the weight (*G* [kg]) of the stone shifts horizontally and thus also the arm with the line of rotation (*A*) [fig. 10].

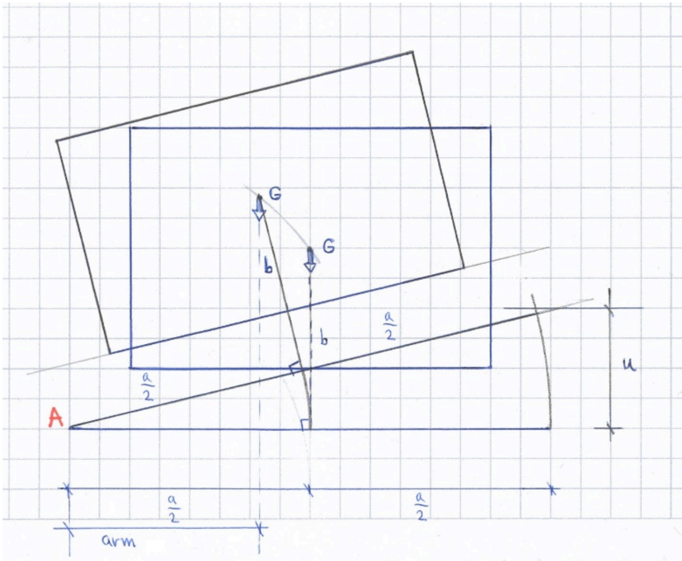
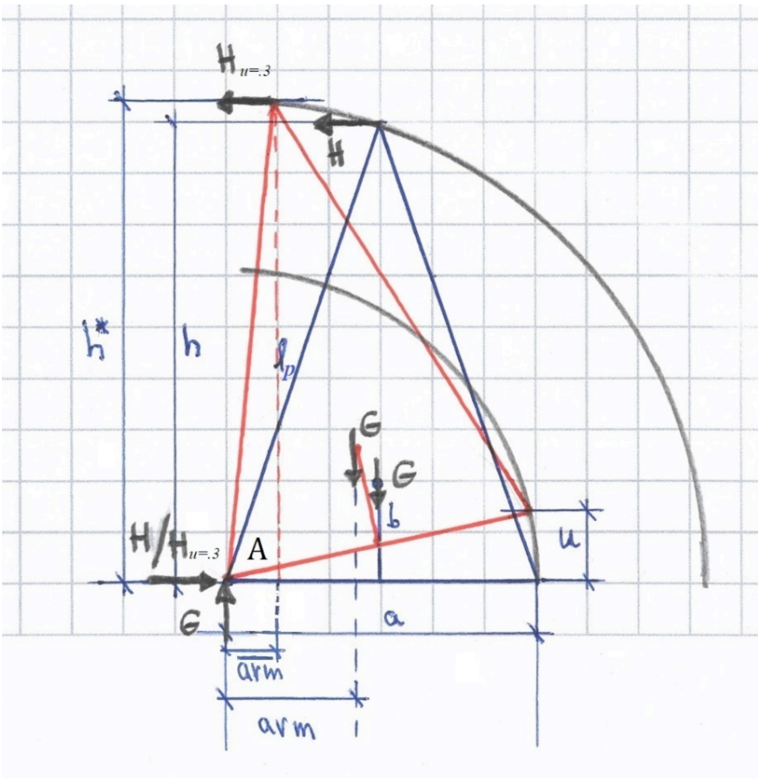


Fig. 10. Tilting a stone.

The tilting formula, for which the appendix at the end of the paper will provide the additional background, describes the process of tilting. In each step of tilting there is equilibrium [fig. 11]. The only constant is the weight of the stone *G*, the weight of the tilt levering cage will be neglected as it only contributes less than 10% of the total weight. The *arm*, the horizontal pulling force *H* and the height of the top of the leverage cage *h** will all vary during the course of tilting. Thus, the expression of *H* will be non-linear [fig. 11-12]. (l_p is the length of the pole in the horizontal projection of the tilt levering cage, which is practically equal to h, fig. 11).



$$l_p^2 = \overline{arm}^2 + h^{*2} \Rightarrow h^{*2} = (l_p^2 - \overline{arm}^2)^{1/2}$$

$$H \cdot h^* = G \cdot arm \Rightarrow$$

$$with:\ \overline{arm} = \frac{a(a^2 - u^2)^{1/2} - 2hu}{2a}$$

$$H = \frac{G \cdot arm}{(l_p^2 - \overline{arm}^2)^{1/2}}$$

$$H = \frac{(a(a^2 - u^2)^{1/2} - 2bu)}{((2al_p + a(a^2 - u^2)^{1/2} - 2hu)(2al_p - a(a^2 - u^2)^{1/2} + 2hu))^{1/2}} G$$

$$H_{(u=0)} = 232\,kg : H_{(u=0.3)} = 175kg$$

$$if\ F = H\sqrt{2} \Rightarrow$$

$$F_{(u=0)} = 328\,kg : F_{(u=0.3)} = 247kg$$

Fig. 11. Geometric relation of tilting: the tilting formula.

The required tugging forces *F* would indicate that even when applying modern working regulations of 26 kg/person, a work force of two teams of ten to twelve people would suffice to perform this upward transport. This may be compared positively with the workforce of a thousand haulers required if ramps were used.[38] When tilting the tilt levering cage in each step reached in excess of a vertical of 0.3 metres, the required horizontal pulling force will reduce with about 25% [fig. 12].

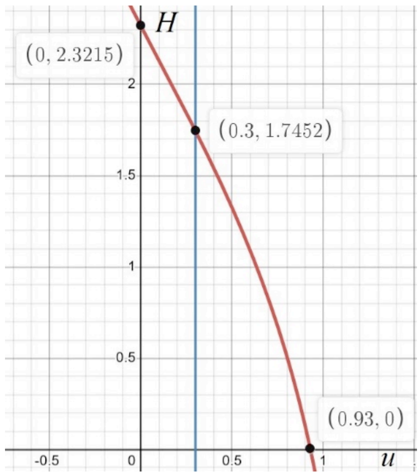


Fig. 12. Relation of *H* and vertical displacement *u*.

It is important that during tilting the tilt levering cage does not slip. The coefficient of friction of wood is about 0.4, so the threshold value that needs to be exceeded before slipping occurs is half the weight of the stone multiplied with the coefficient of friction. With the stone being canted in each step by 0.3 metres the horizontal force remains well below the threshold value [fig. 13]. The ratio between the sliding force of the stone, in the tilt levering cage, and its weight is about 0.2 during tilting, and therefore below the threshold of sliding.

$$F_S \leq \mu F_N$$

$$\mu_{wood} = 0.4$$

$$F_{S,max} = 0.4 * \frac{G}{2} = 0.4 * \frac{2500}{2} = 500\,kg$$

$$\frac{H}{2} \leq F_{S,max} \Rightarrow 232 \leq 500$$

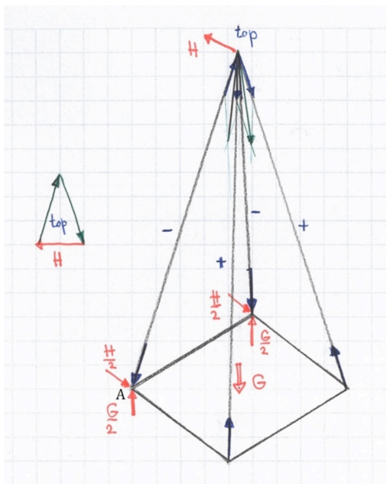


Fig. 13. Threshold of sliding.

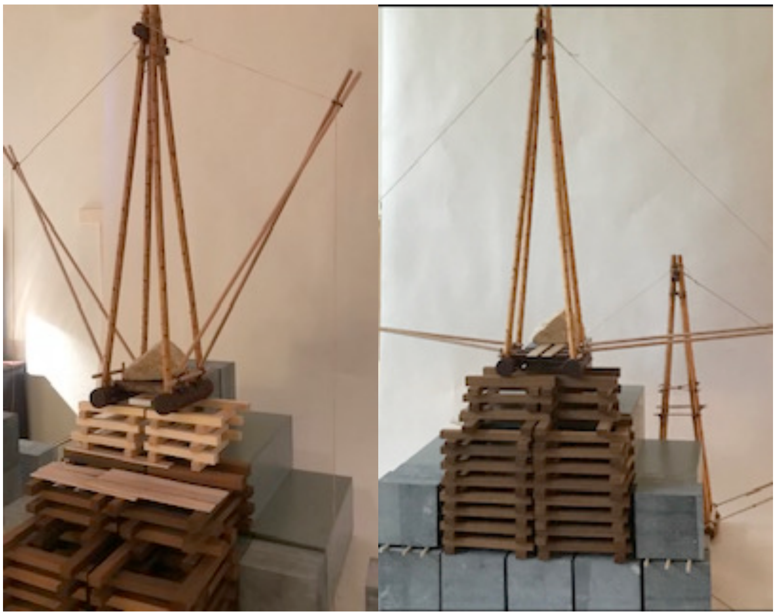
[38] M. LEHNER, Z. HAWASS, *op. cit.*, p 419.

The use of the tilt levering cage

In our proposal we use tilting for lifting the building stones. By pulling the top of a tilt levering cage perpendicular to the girders in the right direction and with sufficient manpower, the tilt levering cage is tilted over one girder. There may be three to four ropes attached to the cage top, each pulled by up to four men.[39] When the tilt levering cage is tilted over one girder, the other girder is lifted, and the thus created space is box cribbed using poles. These poles have approximately the length of a girder. By this box cribbing a broad and stable pillar was formed under the girder [fig. 8]. By subsequent tilting over the other girder, space is created under the first, which again is box cribbed forming a pillar. By repeating this opposite tilting and box cribbing, the tilt levering cage is raised in its entirety, supported by two growing stable box cribs.

In practice, to get the tilt levering cage up the side of a pyramid, it was used in combination with a giant stairway to bring the stones to all required heights.

Pulling the top of the tilt levering cage near a corner or near the top of the pyramid under construction might become difficult, due to a lack of space for the pulling workmen to stand on. To solve this problem one or two cedar side bipods could be used attached to respectively one or both girders. One side bipod could be used when the tilt levering cage was used close to the corner and two side bipods when getting close to the top. Instead of horizontal pulling, workmen could now easily pull down the bipod, standing a few steps below. As can be seen from the force polygons, the side bipod system with the more horizontal position is the most efficient method and requires the less vertical pulling force V_2, as compared with a more vertical side bipod (: $V_2 < V_1$) [fig. 14].



[39] As shown in Fig. 11, a total of 10 men at each side would easily suffice.

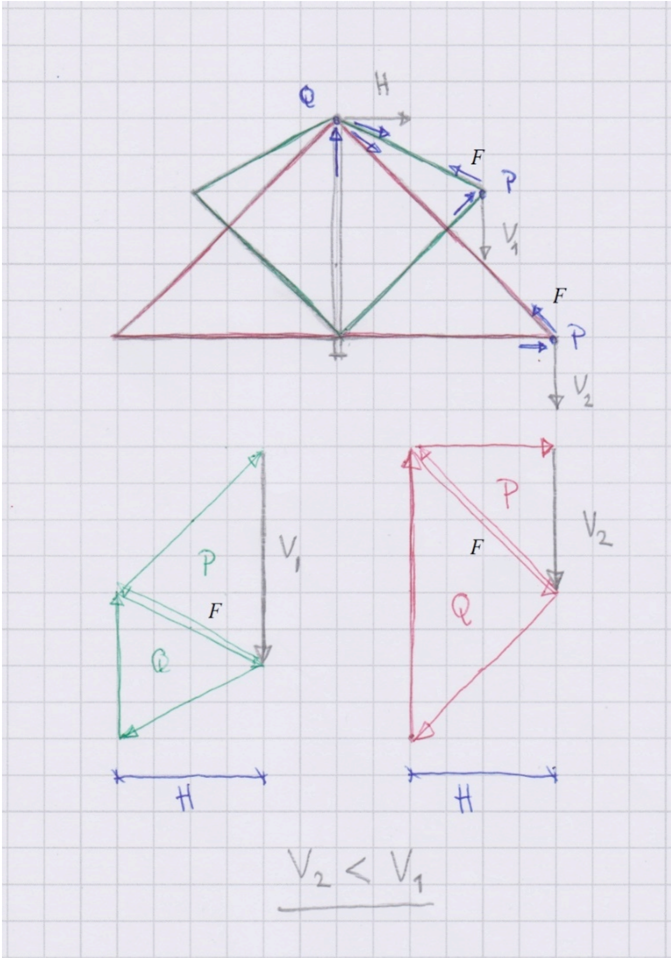


Fig. 14. Side bipods in two different positions and their respective pulling forces: $V_2 < V_1$.

These above-mentioned steps of controlled tilting the tilt levering cage combined with box cribbing as propping, was repeated during the entire process of lifting stones to build the pyramid. The first stone layer (the entire base of the pyramid) was created, after leveling the construction site as needed. The stones on sledges were moved and steered horizontally by manpower using mud, rounded desert sand or roller poles.[40] The poles used for box cribbing could probably have a double function as roller poles as well.

For the construction of the second stone layer, a tilt levering cage was used for the vertical transport to bring the stones onto the first stone layer. For this, the tilt levering cage was placed against the first stone layer. When the bottom of the stone in the tilt levering cage had reached the height of the first stone layer, the stone was horizontally offloaded. Thereafter the stone was transported over the first layer to its final position to form the entire second stone layer.

When the second layer of stones was positioned in its entirety, the run (approximately 1 metre) of the first step of the pyramid was not deep enough to place the tilt levering cage. To

[40] M. ISLER, *op. cit.*, p. 242-243. On the use of water when sliding objects over sand, cf. A. FALL, B. WEBER, M. PAKOUR *et al.*, *Sliding Friction on Wet and Dry Sand*, *Physical review letters* 112, 2014, p. 175502.1-4.

solve this problem of too little space, a few stones were left out from the first stone layer, interrupting the step, thus creating a niche in the stone layer. The niche was as deep as the first run and thereby upwards flush with the rise of the first step. The niche had a width enough for the two side by side growing box cribs for the support of the tilt levering cage.[41] The tilt levering cage was placed in the middle of this niche and the process of lifting could start.

During lifting the tilt levering cage passed the first stone layer and lifting went on as far as necessary to offload the stone horizontally onto the second stone layer to construct the third[42] [fig. 15].

Fig. 15. A tilt levering cage on his way passing the second layer, and one standing upon the third.

For building the fourth stone layer a loaded tilt levering cage was lifted and, in its entirety, offloaded onto the second stone layer, directly in another constructed niche. This second niche was constructed in the third stone layer [fig. 15]. This niche doubled the run of the second step, which meant enough space would be available for the tilt levering cage. This niche was just like the one constructed in the first stone layer, and it was situated exactly in line above the first.

From this niche the fourth stone layer could be reached with the tilt levering cage.

By repeating this process of constructing niches, every other step, up the slope of the pyramid all stone layers could be reached with the tilt levering cage. From some distance the line of successive niches looked like a giant stairway. A video may show this process clearly (https://vimeo.com/660508973, code: hugb). Depending on the situation, the niches could be

[41] In this example, sufficient stones (approximately 1.27 - 1.27 - 0.8m per piece, see fig 1) were left out initially to create the at least 3 metres wide niche. These were placed directly next to the niche, which required a total width per niche of approximately 6-8 m.

[42] In the next and all higher positions of layers, the necessary space for the tilt levering cage was likewise provided by the space on the run together with the space on the floor in the niche.

built in even or odd stone layers.

The few stones that were left out from the stone layer to create a niche were temporarily placed to the right and left next to the niche on the same run in such a way that it formed a whole with the entrance of the niche at both sides [fig. 15]. Niches were left open until the final vertical transport with the tilt levering cages would be completed.

The final activity with the tilt levering cage was transporting upwards the limestone for the outer stone layer, the casing of white Tura limestone. This casing was applied from the top downwards. When the topmost niche of a giant stairway was reached in this downwards building process, the niche was closed, with which the run was restored. This recovery was done by horizontally moving back the stones into the niche that were temporarily placed to the left and right on the run. Thereafter white casing stones were placed covering the whole, while erasing all traces of the former niche. This final building process went on downwards until the base.

At the pyramid under construction, there were series of niches in line that looked like giant stairways. Several of these stairways could run parallel to each other, on all sides, of the pyramid under construction. Many loaded tilt levering cages, at its peak possibly many hundreds,[43] could find their way up. The offloaded tilt levering cages could be lowered in a controlled manner possibly on sledges over a slide, using ropes.

For efficient horizontal transport of the building stones, every single one could be placed on transport sledges,[44] a wooden frame with a flat bottom. These sledges could be dragged over the ground or pulled over roller poles. The building stones were transported on these sledges from the quarries, loaded and offloaded from the tilt levering cage, until their definitive offloading from the sledges at their final position[45] up the pyramid. Many scenes on temple walls show the use of sledges on land and on boats, with heavy loads fixed to them.[46]

Higher up in the pyramid, the building stones have a smaller size and therefore smaller stone layers. The rises of those steps are less high and the runs are less deep. Apart from the manpower the size of the tilt levering cage, the size of the niches and/or the number of steps taken in a flight may be adapted accordingly, but these differences do not influence the principle of the method proposed. The smaller and therefore lighter stones higher in the pyramid would make horizontal towing and vertical transport with a tilt levering cage easier for the workers.

The largest stones that were used inside the structure of the Great Pyramid for lining inner spaces and as roofing blocks for the burial chamber could also be lifted using the method described above. An increase in numbers and scale was applied for this. So, in case of very large stones several wooden truss levers could be constructed and fixed to the stone itself. The

[43] Every stairway probably needed approximately 30m of space in width to work easily. That means 4x231/30 = 30 places possible for starting a giant stairway at ground level for the Great Pyramid. The average stairway had a length of 60 metres, for raising approximately 50 layers. On these 50 layers 25 giant stairway runs might be in use. If all were filled with a tilt levering cage, 30x25= 750 tilt levering cages might be concomitantly in use for lifting stones. Evidently (therefore 'the average stairway'), stairways that reached the edge (all but four) could not continuously be used, but towards the middle of the sides the stairways became longer. The stairways that went temporarily out of use would be reused during the top down casing process.

[44] M. ISLER, *op. cit.*, p 239.

[45] *Ibid.*, p 289.

[46] M. LEHNER, Z. HAWASS, *op. cit.*, p 414, see also Fig. 2.

stone was not transported in a tilt levering cage but became an integral part of the construction of several connected truss levers in line on top of it. The truss levers were fixed to the stone using poles and wood joints such as pegged notches, pegged mortice and tenon and lashings [fig. 16]. During tilting, propping was done through box cribbing.

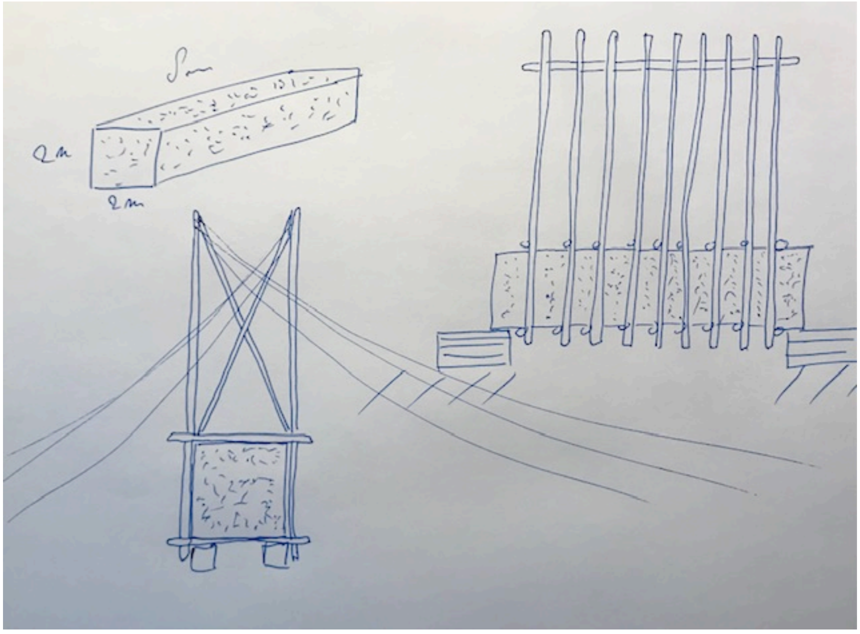


Fig. 16. Artist impression of a Tilt Levering Cage with the stone as an integral part of the tilt levering construction.

The real experiment

The tilting of a stone within a tilt levering cage was successfully, controlled and safely, tested on June 14, 2021. With a tilt levering cage construction as described above, made in full size from pine connected with notches and lashing connections, a weight of 2370 kilograms was levered 7 centimetres in a minute. The necessary pulling power was provided by three persons. With a team of ten men and women, this result was improved on June 19, 2021. In approximately 4 minutes, on the first attempt, we levered a weight of 2370 kilograms vertically by 40 cm [fig. 17]. We have shown with this experiment and a successful experiment on scale 1:10 with niches that the principle of lifting with a tilt levering cage does work.

Conclusion

This hypothesis on how the stones could have been moved vertically is compatible with what we know of wood import over sea from the Levant, as well as available techniques and achievements at that time. In our opinion, the use of a tilt levering cage combined with propping through box cribbing could be the creative solution that enabled the Egyptians to build pyramids without the need to construct extensive ramps. It also provides an explanation for the description of the building practice as recorded by Herodotus, which has remained unexplained thus far. Due to its size, the tilt levering cage would require more space than available on a single run. For that, a series of temporary niches had to be created, which were filled with stones after the work was finished. The niches resembled a giant stairway. The combination of a tilt levering cage, box cribbing, the giant stairway formed by niches with the required pulling forces, made it possible to reach all levels of the pyramid, including the top for placing the pyramidion.

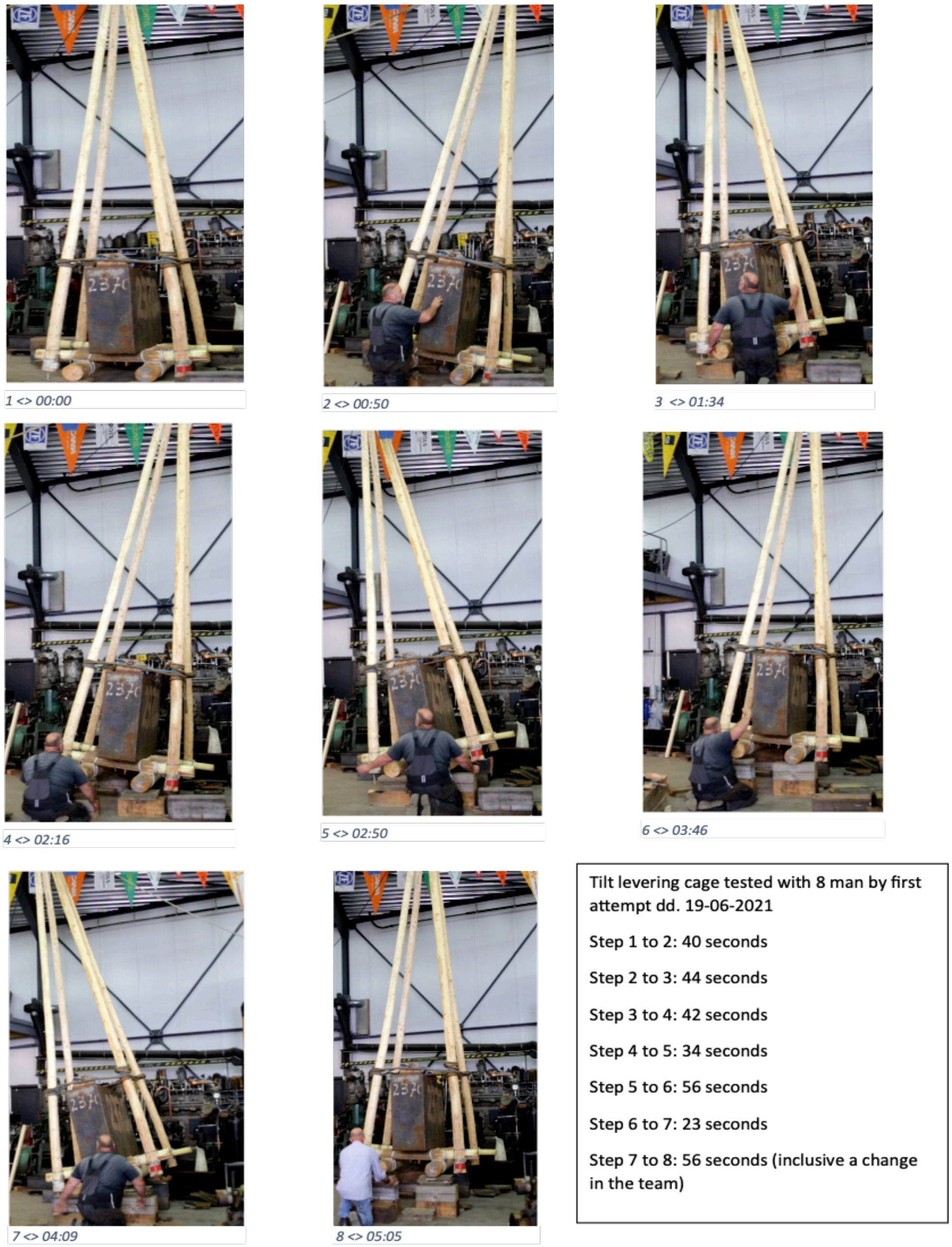


Fig. 17. The first and successful attempt levering a 2370 kg stone with a tilt levering cage.

Appendix

Background of the derivation of the tilting formula

The geometric relation for tilting the cage over line *A* and the horizontal displacement *v* [m] and vertical displacement *u* [m] is presented. By rotating the tilt levering cage over line *A* the movement forms part of the arc of a circle, in which the width of the cage *a* [m] is its radius. As can be seen [fig. 18], rotating the cage also involves a horizontal displacement *v* along the x-axis which is perpendicular to the axis of rotation. This is important because it means that the centre of gravity of the stone will also shift horizontally. This influences the *arm* [m] and thus the horizontal force *H*.

Pythagoras' theorem:

$$(a-v)^2 + u^2 = a^2$$

$$\Rightarrow v = a - (a^2 - u^2)^{1/2}$$

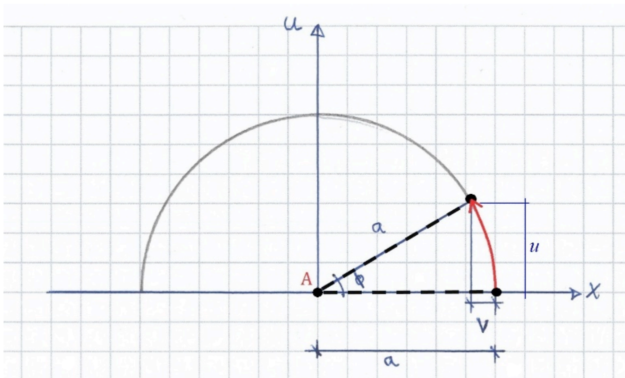
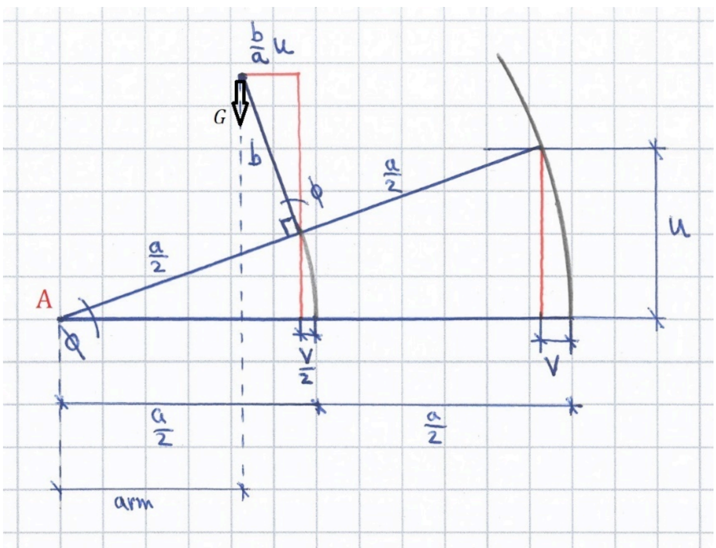


Fig. 18. Principle of rotation.

The horizontal displacement *v* is expressed in the width *a* of the cage and the vertical displacement *u* [fig. 18]. This means that the *arm* can be solely expressed in cage width *a* and vertical displacement *u*, note relation is nonlinear which is the result of the centre of gravity of the stone shifting horizontally during the lilting [fig. 19].



$$\frac{u}{a} = \frac{\frac{bu}{a}}{b}$$

$$arm = \frac{a}{2} - \frac{b}{a}u - \frac{v}{2}$$

$$with: v = a - (a^2 - u^2)^{1/2}$$

$$\Rightarrow \frac{a}{2} - \frac{b}{a}u - \frac{a - (a^2 - u^2)^{1/2}}{2}$$

$$arm = \frac{a(a^2 - u^2)^{1/2} - 2bu}{2a}$$

$$u_{(arm=0)} = \frac{a^2}{(a^2 + 4b^2)^{1/2}}$$

Fig. 19. Geometric relation of *arm*.

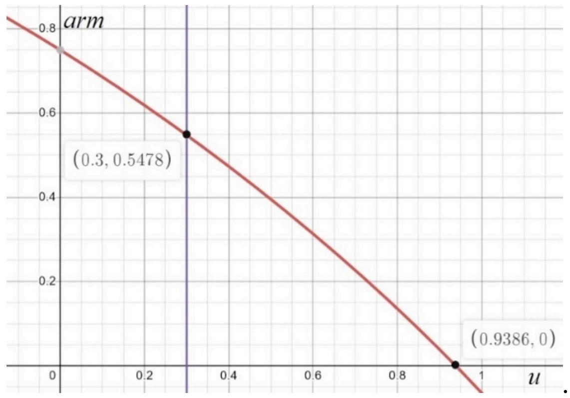


Fig. 20. Relation of *arm* and vertical displacement *u*.

If the *arm* becomes zero through the extent of tilting u ($_{arm\,=\,0}$), the centre of gravity will be exactly above rotation point *A*, thus the horizontal force *H* will become zero. The entire levering cage will be just in equilibrium, which confirms the correctness of the derivation.

The expression of the *arm* leads to the derivation of the tilting formula as seen in see fig. 11. The tilting formula is the expression of the horizontal pulling force *H*, which is a function of the dimensions of the tilt levering cage and the upward displacement *u*.

Whilst tilting the tilt levering cage, the centre of gravity of the weight of the block moves towards the rotation point *A*, the vertical support reaction in *A* remains the same, notably the weight of the block *G*. But the load distribution of the stone within the leverage cage alters during tilting. This can be shown by taking the sum of the moments around rotation point *A* equal to zero so that in each step the tilt levering cage is in equilibrium and determining vertical forces which act on the girders V_r and V_l [kg] [fig. 21].

sum of the moments
around rotation point A:

$$\Sigma M_A = 0:$$

$$G \cdot arm - V_r \cdot (a - v) = 0$$

$$with: v = a - (a^2 - u^2)^{1/2}$$

$$arm = \frac{a(a^2 - u^2)^{1/2} - 2bu}{2a}$$

$$\Rightarrow V_r = \frac{a(a^2 - u^2)^{1/2} - 2bu}{2a(a^2 - u^2)^{1/2}} G$$

$$vertical\ equilbirium: V_l = G - V_r$$

$$\Rightarrow V_l = \frac{a(a^2 - u^2)^{1/2} + 2bu}{2a(a^2 - u^2)^{1/2}} G$$

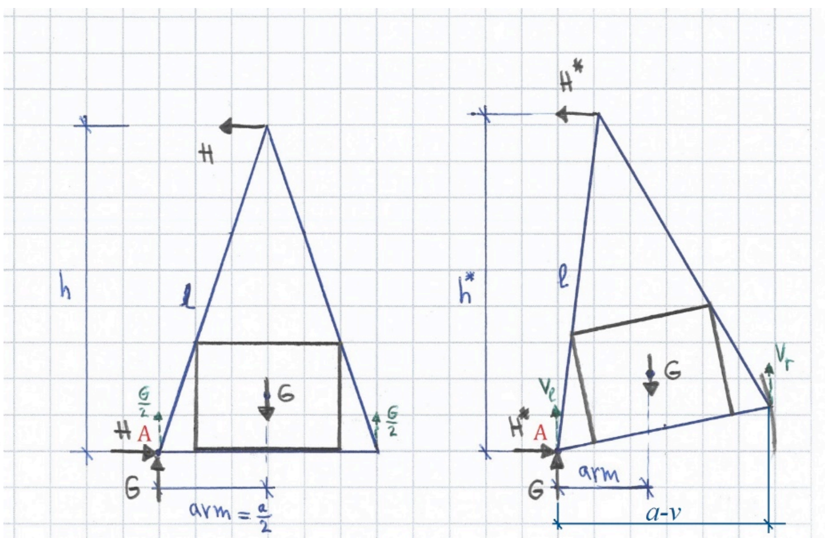


Fig. 21. Load distribution in the tilt levering cage during the course of tilting.

When the centre of gravity is above point *A*, there is no longer a state of load distribution needed via the means of the tilt levering cage [fig. 22].

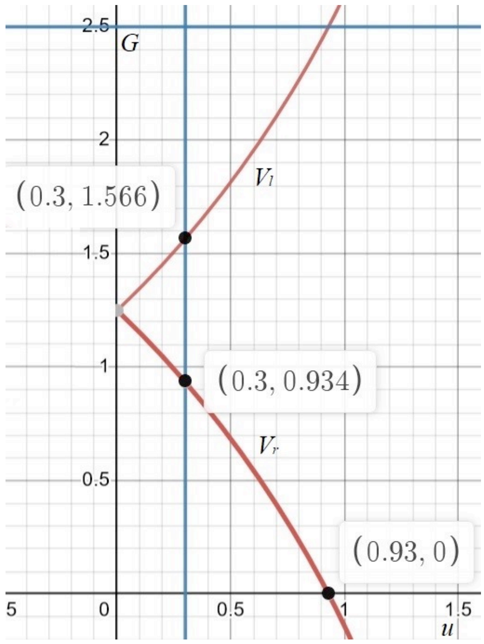


Fig. 22 Graph of load distribution in the tilt levering cage during the course of tilting.

Acknowledgements

The authors wish to thank Mr Schimmelpenninck van der Oije for initial Egyptological help; Mr Pit and Mr Romke for technical help in making the tilt levering cage; Mr Bik for critical technical reading; Mrs Bik-Huls for Greek linguistical help; Mr Slagt for making the models and Mr Frank for making the stop motion pictures using these models.