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THE GREAT PYRAMID was built by King Khufu in the 26th century BC. The main chamber within is the King's Chamber, which is a rectangular room that is empty apart from a lidless, granite sarcophagus. Its walls are comprised of 100 granite bricks and within this brickwork there are three openings. One is the entrance way, the other two are shaft inlets. In combination, the sarcophagus, the inlets, and the doorway have been termed the 'features'. These features are relatively low down in the room and they all terminate at the same height, suggesting that the builders were drawing attention to this singular height. According to Gantenbrink¹ the southern shaft inlet is the start of a long shaft that is, for the majority of its length, angled at 45°. At this location, in 26th century BC Egypt, the maximum elevation that the sun would attain in the year was achieved at noon on the summer solstice when it reached 84°; while at the winter solstice it reached its annual noonday low of 36.05°.² It is my hypothesis that the architects designed the King's Chamber so that its floor corresponded to the winter solstice low for the noonday sun; the ceiling corresponded to the summer solstice high, and the height of the features was commensurate with the 45° sun. In this manner, the design of the room would be of an architectural calendar that tracked the rise and fall of the sun through the year.

Solar alignment

The shaft inlets in the King's Chamber are the start of two long shafts that presently exit the exterior of the pyramid, however, it is unknown if they continued on through the layer of stones that originally encased the pyramid. In Petrie's opinion,³ "it is almost certain that they similarly continued to the outside". In 1881, Petrie climbed the Great pyramid so that he could check the angle of the southern shaft using the sun's elevation:⁴ "These details were seen by examining it with a telescope on Feb. 8, and by photographing it on Nov 2, 1881; these being the days on which the sun shines down it at noon." In the 26th century BC, the sun's elevation as it crossed the meridian at the winter solstice was 36.05°, and it was 84° at the summer solstice.⁵ 45° is at the lower end of this range. The King's Chamber is a tall room

¹ R. GANTENBRINK, *The Upuaut Project; A Report by Rudolf Gantenbrink*

http://www.cheops.org/startpage/thefindings/thefindings.htm (1 June 2020).

² NASA JET PROPULSION LABORATORY, *Horizons web-interface*

https://ssd.jpl.nasa.gov/horizons.cgi#top accédé (2 March 2021).

³ W.M.F PETRIE, *The Pyramids and Temples of Gizeh*, London, 1883, p. 52. ⁴ *Ibid.*, 84.

⁵ NASA JET PROPULSION LABORATORY, op. cit.

and the southern shaft inlet is placed at a relatively low height in the chamber. This raises the prospect that the design of this room was intended to reflect the solar range, with the placement of this inlet preserving the elevation of the sun when it was commensurate with the inclination of the southern shaft (i.e. 45°).

Meridional transits

Since the values in question here are solar elevations for the sun as it crossed the meridian, then it must be shown that the Ancient Egyptians had some history of measuring astronomical bodies as they crossed the meridian. It is well known that they had star clocks which operated using the heliacal risings of certain stars, however, these involved the first appearance of stars above the horizon in the pre-dawn. Such clocks involved looking east, rather than due-south.

Two versions of a meridional system have come to light. Evidence of 'Transit Decanal Clocks' are found in the Book of Nut.⁶ Also, 'Ramesside Star Clocks' are known to have been employed.⁷ Both gauged time by the transit of stars across the meridian. An example of an instrument used for this purpose is the 'Tutankhamun Transit Instrument.'⁸ Wall⁹ however, points out that there is no evidence of such observations in the Old Kingdom and he is correct insofar as no such instruments have been recovered from that era to date. Architecture from the era though, does suggest meridian transits were important. The north facing entrances of a typical Old Kingdom pyramid targeted the circumpolar stars as they crossed the northern meridian¹⁰. Furthermore, the north and south facing boat pits on the east side of the Great Pyramid are interpreted by Hassan¹¹ as solar barques which were designed to enable the soul of the deceased to sail north and south as the sun boat was believed to do. According to $\check{C}ern \check{y}^{12}$ the four boat pits on the east and south side of the complex were designed to carry the king to the cardinal points. The precise alignment of the pyramid to the cardinal points reiterates an obsession with cardinality and it is only natural for this obsession to have extended to the stars themselves, especially when most believe that the Great Pyramid was oriented by means of observing the circumpolar stars at their culmination.¹³

Evidence of solar worship in Dynasty 4

For the hypothesis that solar data was incorporated into the design of the King's Chamber to be tenable, it requires proof of the importance of the sun to Dynasty 4 Egyptians. The importance of the sun to kings of Dynasty 5 and 6 is clearly evidenced in the Pyramid Texts.

⁶ M. CLAGETT, Ancient Egyptian Science II. Calendars, Clocks and Astronomy, Philadelphia, 1995, p. 56-57. ⁷ *Ibid.*, p. 59-60.

⁸ G.E. HALE, Beyond the Milky Way, New York, 1926, p. 6-8.

⁹ J. WALL, "The Star Alignment Hypothesis," JHA 38/2, p. 199-206.

¹⁰ I.E. EDWARDS, "The Air-Channels of Chephren's Pyramid," in W. Simpson, W. Davis (eds.), Studies in Ancient Egypt, the Aegean, and the Sudan: Essays in Honor of Dows Dunham on the Occasion of his 90th Birthday, Boston, 1981, p. 56.

¹¹ S. HASSAN, The Great Pyramid of Khufu and its Mortuary Chapel with Names and Titles of Vols. I-X of the *Excavations at Giza* VI. Cairo, 1960, p. 46-47. ¹² J. ČERNÝ, "A Note on the Recently Discovered Boat of Cheops," *JEA* 41, 1995, p. 77-78.

¹³ A. POGO, "The Astronomical Ceiling Decoration in the Tomb of Senmut," Isis 14, 1930, p. 301-325; K. SPENCE, "Ancient Egyptian Chronology and the Astronomical Orientation of Pyramids," Nature 408, 2000, p. 320-324; J. BELMONTE, "On the Orientation of Old Kingdom Egyptian Pyramids," JHA, Archaeoastronomy Supplement 32, 2001, p. 11.

The ascension of the king to the solar barque is evidenced in Pyramid Text 461a: "N. ascends to heaven, to thee, O $R\bar{e}$ '."¹⁴ It is also evidenced in PT 1171a-b: "Pure one, assume thy throne in the boat of $R\bar{e}$ ', that thou mayest sail the sky."¹⁵ While these Pyramid Texts belong to Dynasty 5 and 6, Re worship may have reached new heights during Khufu's reign. According to Stadelmann¹⁶ Khufu was the first king to be identified with Re, citing as evidence that Khufu's sons 'Djedefre' and 'Khafre' were the first kings to carry the epithet "son of Re," thus identifying their father with Re.

There is also evidence of solar data being preserved in architecture from Dynasty 4. King Snefru preceded King Khufu and he constructed a pyramid complex at Dahshur dominated by the so-called 'Bent Pyramid' and the 'Red Pyramid.' In Nuzzolo's opinion¹⁷ "the Dahshur pyramids were mainly connected to the sun cult and the concept of the horizon as the place for the accomplishment of the sun cycle." According to Lehner¹⁸ the Bent Pyramid "probably had more to do with the solar cult and deification of the king than with any attempt at preserving his body." The solar nature of the Bent Pyramid is certainly suggested by the length of its sides. Petrie's survey¹⁹ determined the mean length of the sides to be 189.46 m, which he interprets as "360 cubits of 20.72 +/- .006, the ordinary Egyptian cubit." According to Plutarch's account²⁰ the Egyptian calendar was originally 360 days long, then Thoth waged a bet with the moon and won five extra days. According to Kahl²¹ this was the form of the civil calendar from at least the time of King Zoser. According to Hornung, Krauss and Warburton²² these five days were often disregarded:

"A tendency to regard the year as amounting to only 360 days is evident, for example when the daily income of a temple is stated to be one 360th of the yearly revenue. The well known disregard of the epagomenai in calendar schemes seems to be another consequence of this tendency".

Thus we can see that the number of cubits comprising the length of the base of the Bent Pyramid matched the number of days in the year (excluding the five epagomenal days). There was another structure dating to the New Kingdom which used the same system wherein each cubit represented one day, evident in Diodorus' account²³ of the tomb of King Ramses II: "… there is a circular border of gold crowning the monument, three hundred and sixty-five cubits in circumference and one cubit thick; upon this the days of the year are inscribed, one in each cubit of length…"

Lehner²⁴ argued that the Sphinx Temple and the pyramids of Khufu and Khafre were laid out in such a way that they created a solar phenomenon on the summer solstice, noting that from

http://www.enim-egyptologie.fr

¹⁴ S. MERCER, *The Pyramid Texts*, Toronto, 1952, p. 101,

¹⁵ *Ibid.*, p. 196.

¹⁶ R. STADELMANN, *Die Ägyptischen Pyramiden vom Ziegelbau zum Weltwunder*, Darmstadt, 1985, p. 126.

¹⁷ M. NUZZOLO, "The Bent Pyramid of Snefru at Dahshur. A Project Failure or an Intentional Architectural Framework?," *SAK* 44, p. 271.

¹⁸ M. LEHNER, *The Complete Pyramids*, New York, 1997, p. 129.

¹⁹ W.M.F. PETRIE, *A Season in Egypt*, London, 1887, p. 28, 31.

²⁰ PLUTARCH, *Moralia* (F. Babbitt trans.), Cambridge MA, 1936, V.12.

²¹ J. KAHL, N. KLOTH, U. ZIMMERMANN, *Die Inschriften der 3. Dynastie: eine Bestandsaufnahme*, Wiesbaden, 1995, p. 70-71.

²² E. HORNUNG, R. KRAUSS, D.A. WARBURTON, Ancient Egyptian Chronology, Leiden, 2006, p. 47.

²³ DIODORUS, *Library of History* (C. Oldfather trans.), Cambridge, MA, 1933, I.49.5-6.

²⁴ M. LEHNER, "Giza, A Contextual Approach to the Pyramids," *AfO* 32, p. 141.

the eastern niche of the Sphinx Temple "the sun sets almost exactly midway between the Khufu and Khafre pyramids, thus construing the image of the akhet, «horizon», hieroglyph."

Khufu's pyramid complex contains four boat pits lying just outside and parallel to, the temenos walls. There are two on the south side and two on the east side, each facing a different cardinal point²⁵. Hawass²⁶ argued that the two southern boats were solar boats for the king so that he could "accompany the god Re on his daily trips across the sky." According to Hassan²⁷ these two boats were "a purely solar conception concerned entirely with the daily journey from east to west and the nighty journey from west to east." The presence of solar boat pits around Khufu's pyramid links this king to the sun, however, it is likely that this link is much older than Dynasty 4. According to Emery²⁸ the Dynasty 1 boat pit of King Hor-aha was a solar boat, an identification which Hawass²⁹ states as "almost certain."

Others have theorised that the interior design of the Great Pyramid also carried a solar function. According to Badawy³⁰ the southern shaft was designed to align with the sun to assist with the ascent of the king's soul to the sun. Conman³¹ also argued that the southern shaft carried a solar purpose, linking the sun to the northern skies, thus functioning as a "magical irrigation system." According to Hawass³² this shaft had a specifically solar purpose:

"... it is my opinion that the function of the southern shaft ... is linked with the function of the two solar boats buried on the southern side of the Great Pyramid It is thought that the king would use the solar boats to accompany the god Re on his daily trips across the sky."

Against this, Gantenbrink³³ argued that because the shafts do not provide a direct site line, they could not function in such a manner. Starting from the chamber, they all carry the form of a short horizontal section, followed by an incline. Edwards³⁴ counters this by noting that the form of the shafts replicates the pattern typically found in pyramid design, wherein a horizontal section leads from the burial chamber to an inclined entrance shaft:

"In this respect their design conforms with that of the regular entrance-corridors of pyramids, one of whose functions is also believed to have been to serve as a symbolical channel of approach to the circumpolar stars."

Another theory is that the King's Chamber shafts had stellar targets. The southern shaft inclines at 45° and Trimble³⁵ identified the stars culminating at this elevation in Khufu's era

²⁵ S. HASSAN, *op. cit*, p. 66-68.

²⁶ Z. HAWASS, *Magic of the Pyramids: My adventures in Archeology*, Montevarchi, 2015, p. 105.

²⁷ S. HASSAN, *op. cit*, p. 46.

²⁸ W.B. EMERY, Excavations at Sakkara 1937-1938: Hor-aha, Cairo, 1939, p. 8.

²⁹ Z. HAWASS, The Funerary Establishments of Khufu, Khafra and Menkaura during the Old Kingdom, Ph.D. Thesis, Department of Oriental Studies, University of Pennsylvania, 1987, p. 38-39.

³⁰ A. BADAWY, "The Stellar Destiny of Pharaoh and the so-called Air-Shafts of Cheops Pyramid," MDAIK 10/2-3, 1964, p. 189-206. ³¹ J. CONMAN, "Speculation on Special Sunlight and the Origin of the *wš3w* Hour," *AdE* 3, p. 7.

³² Z. HAWASS, Magic of the Pyramids: My adventures in Archeology, p. 105.

³³ R. GANTENBRINK, op. cit.

³⁴ I.E. EDWARDS, op. cit., p. 56.

³⁵ V. TRIMBLE, "Astronomical Investigation Concerning the So-Called Air-Shafts of Cheops' Pyramid," MIO 10, 1964, p. 186-187.

as the Belt stars of Orion. However, a problem exists with this stellar target as it is unknown if these stars were important to the kings of Dynasty 4. The Belt stars belong to the constellation which is presently called 'Orion,' a set of stars which the Egyptians identified with the god Osiris from at least the Middle Kingdom, but earlier than that it is unclear if these stars belonged to this god. Osiris is well-attested to in the Pyramid Texts of Dynasty 5 and 6 (e.g. Uts 436-442, 534, 577), however, it is unknown whether Osiris existed as early as Dynasty 4 (when the Great Pyramid was constructed). According to Shalomi-Hen, Baer, and Posener-Kriéger,³⁶ Osiris appeared for the first time in Dynasty 5 temples.

Vyse³⁷ cleared the blocked southern shaft in 1837 and the release of cool air into the chamber led him to conclude that the purpose of the shafts was ventilation, however, in 1872 shafts were discovered in the Queen's Chamber which do not exit the exterior of the pyramid, leading many to abandon this theory. Legon³⁸ however, still supports ventilation, citing geometrical harmonies to support this conclusion.

While there is speculation as to the purpose of the southern shaft, the fact remains that this shaft creates an alignment with the sun on two dates in every year. According to Gantenbrin³⁹ the southern shaft of the Great Pyramid is inclined at 45° and runs approximately due-south. Through the year, in 26th century BC Giza, the sun's maximum elevation was 84°, attained at noon on the summer solstice. Its minimum elevation was 36.05°, attained at noon six months later at the winter solstice.

Determining the height of the southern shaft

To determine if this chamber was designed to preserve the sun's range in its vertical arrangement, the sun's maximum elevation needs to be equated with the ceiling of the chamber and its minimum elevation equated with the floor, to then determine if the shaft inlet is located at a height commensurate with the 45° solar elevation. According to Petrie⁴¹ the height of the King's Chamber is 5.8443 m. The floor is of course 0 m. The third measure is the terminus height of the features. Determining this height is difficult on account of the floor being independent of the walls and not being even over the span of the room.

Presently, the southern shaft inlet is not in its original state. It has been deformed, presumably by treasure hunters. Its original form can be discerned from a photo taken by the Edgar brothers;⁴² it was a rectangular inlet that terminated at the brick course line (as shown in Figure 1). This shaft therefore paralleled the northern one in shape, as well as the height that it terminated at (i.e. the base of the second course of bricks formed the ceiling of both shafts).

³⁶ R. SHALOMI-HEN, "The Dawn of Osiris and the Dusk of the Sun-Temples: Religious History at the End of the Fifth Dynasty," in P. Manuelian, T. Schneider (eds.), *Towards a New History for the Egyptian Old Kingdom; Perspectives on the Pyramid Age*, Leiden, 2015, p. 462; K. BAER, *Rank and Title in the Old Kingdom; The Structure of the Egyptian Administration in the Fifth and Sixth Dynasties*, Chicago, 1960, p. 297; P. POSENER-KRIÉGER, *Les Archives du Temple Funéraire de Néferirkarê-Kakaï* II, Cairo, 1976, p. 52-55.

³⁷ R.W.H. VYSE, J.S. PERRING, *Operations carried on at the pyramids of Gizeh in 1837* I, London, 1840, p. 286-287.

³⁸ J.A.R. LEGON, "The Orion Correlation and Air-Shaft Theories," *DE* 33, 1995, p. 45-56.

³⁹ R. GANTENBRINK, *op. cit.*

⁴⁰ NASA JET PROPULSION LABORATORY, *op. cit.*

⁴¹ W.M.F. PETRIE, *The Pyramids and Temples of Gizeh*, p. 258 (pl. xiii).

⁴² J. EDGAR, M. EDGAR, *The Great Pyramid Passages and Chambers*, Glasgow, 1910, p. 248 (pl. cxxiii).

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Fig. 1. From left to right: the northern shaft inlet of the King's chamber (photo by the author); the desecrated southern shaft inlet (photo by the Edgar brothers); and the southern shaft inlet as it probably looked originally (Edgar brothers photo altered by the author).

Petrie⁴³ estimated the height of this course line of bricks above the floor:

"... the floor of the chamber, as is well known, is quite disconnected from the walls, and stands somewhat above the base of the lowest course. It is very irregular in its level, not only absolutely, but even in relation to the courses; its depth below the first course joint varying 2.29 [inches], from 42.94 to 40.65."

Gantenbrink⁴⁴ estimated the southern shaft inlet to be 14 cm high. Applying Petrie's range to this value locates the base of the inlet between 89.255-95.072 cm above the floor, and the original ceiling of the inlet between 103.251-109.068 cm above the floor.

Determining the terminus height for the features

An issue that needs to be addressed before calculating the accuracy of this hypothesis, is determining which height of the inlet should be used to represent the 45° mark? Should it be the upper lip, the bottom ledge, or the mid-point of the inlet?

The shafts terminate at the height that the first course of bricks meets the second course. The doorway is another feature and it also terminates at the same course line. For all three, the second course of bricks forms the ceiling. The fourth feature is the granite sarcophagus which is freestanding, thus its height is independent of the uneven floor. This coffer was rigorously measured by Petrie⁴⁵ who determined its mean height to be to be 104.927 cm. This height sits within the range of heights that Petrie provided for the course line (103.251-109.068 cm). Since it appears that everything of note within this chamber was designed to terminate at one singular height, and since the sarcophagus is the only feature independent of the floor, then it is likely that its height is indicative of the intended height for the course line. Adding to the likelihood of this is the Ancient Egyptian measure that the height of the sarcophagus equates to. They used the Royal Cubit. While its value varied, Petrie⁴⁶ estimated the value of the cubit in use in the King's Chamber to be 52.405 cm (KCC). Two such cubits are 104.81 cm, which

⁴³ W.M.F. PETRIE, *op. cit*, p. 82.

⁴⁴ R. GANTENBRINK, *op. cit.*

⁴⁵ W.M.F. PETRIE, *op. cit*, p. 86.

⁴⁶ W.M.F. PETRIE, *op. cit*, p. 81.

is only 1.17 mm off Petrie's estimate for the height of the sarcophagus. Smyth 47 also measured the sarcophagus and determined its mean height to be 104.826 cm, which carries an error from two KCC of 0.16 mm. The intended height of the sarcophagus appears to be a round number of two KCC. The preference for round numbers is evidenced elsewhere in this pyramid: the floor of this chamber measures 10 x 20 cubits; the pyramid's base and height are 280 and 440 cubits respectively⁴⁸.

The exact value of the cubit in use has two potential values within the King's Chamber: the sarcophagus cubit (52.464 cm) and the KCC (52.405 cm). The KCC cubit was considered by Petrie⁴⁹ to be the most accurate source for the cubit in the Great Pyramid:

"Probably the base of the chamber was the part most carefully adjusted and set out; and hence the original value of the cubit used can be most accurately recovered from that part. The four sides there yield a mean value of $20.632 \pm .004$, and this is certainly the best determination of the cubit that we can hope for from the Great Pyramid."

Consequently, that is the cubit that I will use in this article.

Determining whether the King's Chamber contains solar data in its vertical arrangement

This creates the following data set for the key heights within the King's Chamber: floor = 0 cm = 0 cubits; features = 104.810 cm = 2 cubits; ceiling = 584.429 cm = 11.152 cubits.

The apparent solar elevations which these heights need to pair with are as follows: winter solstice nadir = 36.05° ; elevation targeted by the southern shaft = 45° ; summer solstice zenith = 84° .

For the hypothesis presented in this paper to be resolved, these values would need to correspond mathematically. In other words, in combination, the floor of the chamber would need to match the sun's winter solstice noon-day low, the height of the features would need to match the sun's elevation when it aligned with the southern shaft, and the height of the ceiling would need to match the sun's summer solstice noon-day high.

A simple way of solving this equation is to equate the ceiling of the chamber to 84° and the features to 45° , and then see what estimate the floor would yield for the winter solstice solar elevation.

The vertical distance between the ceiling and the features is 9.152 cubits which would need to equate to 39° of solar elevation. Dividing 39 by 9.152 yields the number of degrees that each cubit would represent: 4.261°. Doubling this value yields how many degrees the height of the features would represent: 8.523°. Subtracting this amount from 45° yields the chamber's estimate for the winter solstice elevation for the sun: 36.477°. This value carries an error of 0.427° from NASA's estimate for the sun's apparent minimum noonday value for the 26th

⁴⁷ C.P. SMYTH, Our Inheritance in the Great Pyramid: Including all the Most Important Discoveries up to the Present Time, London, 1874, p. 138.

⁴⁸ W.M.F. PETRIE, *op. cit*, p. 182-183.

⁴⁹ *Ibid.*, p. 81.

century BC⁵⁰. Given the technology available at the time for observing and recording solar elevations, as well as inevitable error in building a room with such massive stone blocks, this is an incredibly small margin. The design of this chamber does appear to incorporate data of the sun's range in its layout (as shown in Figure 2).



Fig. 2. Profile of the King's Chamber with the 'features'. Noonday solar elevations at the solstices correspond to the floor and ceiling of the chamber, while the height of the features corresponds to the sun when it reached the midpoint (45°) in the southern skies (drawing by the author).

The Queen's Chamber

If one chamber in the Great Pyramid followed the scheme outlined above, then the other chamber might employ it as well. The so-called 'Queen's Chamber' also contains shafts, however, these are higher up in the room. As is the case in the King's Chamber, they terminate at a course line in the brick work, however, in this chamber, it is where the second course of bricks meets the third. This height is marked throughout the room (just as the height of the features was in the King's Chamber). Set into the eastern wall of the chamber, there is an unusual feature called the 'niche'. It is two cubits deep and rises up in five sections, with each section getting progressively smaller than the last. The main (i.e. lower) section of the niche terminates at the same height as the shaft inlets⁵¹. The doorway also terminates at this height.

Since the height of the features in the King's Chamber was consistent with the angle of the southern shaft in this scheme (i.e. 45°), then the same system might be employed in this chamber; in which case, the height of the features in the Queen's Chamber would represents the angle of its southern shaft.

 $^{^{50}}$ Using the cubit derived from Petrie's measure of the sarcophagus (i.e. 104.927 cm) yields an estimate for the sun's elevation at noon on the winter solstice of 36.466°. This estimate carries an error of 0.416° from the sun's actual elevation.

⁵¹ I am defining the 'main section' as the bottom section of the niche, which many believe was built to house a statue of the king. It is the approximate height of a person and is the largest section of the niche.

Determining whether the Queen's Chamber encodes solar data in its vertical design

Gantenbrink⁵² sent a robot up this shaft in 1992 and he recorded an average angle of inclination for the southern shaft of 39.6°. Assuming that the same system was applied to this chamber, then the floor (set to a height of 0 cm) would equate to the same solar elevation that the King's Chamber floor did, which is 36.473° . However, the working value for the sun's minimum elevation is unlikely to have been such a complex fraction, so I am rounding this value to the nearest tenth of a degree; 36.5° . According to Petrie the Queen's Chamber's features all terminate at 170.536 cm^{53} . If the same system that is potentially in use in the King's Chamber is in use here, then this height should correspond to 39.6° , in which case, 170.536 cm would represent 3.1° . Dividing this height by 3.1 will show how many vertical centimetres each degree within this chamber would represent: $170.536 \text{ cm} \div 3.1 = 55.012 \text{ cm}$.

Within the King's Chamber, the floor, the terminus height for the features, and the height of the ceiling were used to mark key solar elevations. If the same system was employed within this chamber, then something of note should be targeted by the ceiling of the chamber.

Confusing the issue, the Queen's Chamber has two terminus points because its roof is gabled. The walls are vertical up to the height of 468.554 cm, then the roof slopes inwards at $\sim 30^{\circ}$ with the apex of the roof 622.554 cm above the floor⁵⁴. Since each degree is represented by 55.012 cm, then dividing both ceiling heights by this value will yield how many degrees that height would represent in this scheme. Adding the baseline value of 36.5° to this will yield the associated solar elevation.

Apex of roof: 622.554 cm \div 55.012 cm = 11.317°. Adding the baseline value of 36.5° = 47.817°.

Vertical wall: 468.554 cm \div 55.012 cm = 8.517°. Adding the baseline value of $36.5^{\circ} = 45.017^{\circ} \approx 45^{\circ}$.

The apex of the chamber yields an indeterminate elevation, however, the height of the vertical sides of the chamber yields a significant number: 45°. This is the elevation targeted by the features in the King's Chamber. However, there is a structure within this chamber which also terminates at a very similar height and so perhaps it was not the walls targeting this elevation, but rather, it was this feature. Just 1.7 cm below the height where the vertical walls terminate, the niche terminates. According to Petrie⁵⁵ the niche rises to a height of 466.852 cm above the floor of the chamber. Applying the same formula to the niche:

Niche height: 466.852 cm \div 55.012 cm = 8.487°. Adding the baseline value of $36.5^{\circ} = 44.987^{\circ} \approx 45^{\circ}$

⁵² R. GANTENBRINK, *op. cit.*

⁵³ W.M.F. PETRIE, *op. cit.*, p. 70. I am using the niche as the most accurate measure for the height of the features in the Queen's Chamber, since it is the dominant structure in this room.

⁵⁴ *Ibid.*, p. 68-69.

⁵⁵ Ibid., p. 70.

The niche

Various theories have been forwarded regarding the purpose of this niche. Greaves⁵⁶ speculated that the niche was where an "idol might be placed." Petrie⁵⁷ argued that a diorite statue of the king once stood within the niche. According to Lehner⁵⁸ this chamber was a "statue serdab." According to Petrie⁵⁹ "the curiously eccentric niche on the E. wall seems as if intended to mark some distance." According to Smyth⁶⁰ its displacement was precisely one "scientific Pyramid" cubit of 25" (63.5 cm). According to Laboy⁶¹ the niche evidences knowledge of Phi with its placement.

The niche incorporates all three measures at play within this chamber: it commences at floor level (0 cm); the main section of the niche terminates where the second course of bricks meets the third course (170.536 cm); and the completed niche terminates at 466.852 cm. In turn, according to this system, these heights represent solar elevations of 36.5° , 39.6° and 45° .⁶² Therefore, this structure built into the eastern wall of the Queen's Chamber carries the same coding that the arrangement of the King's Chamber does, mathematically corroborating the findings from that chamber. The vertical layout of the King's Chamber has been shown to have mapped the sun's range from its zenith to its nadir, with particular attention paid to the midpoint in the southern skies (45°). The Queen's Chamber utilised the same system, with its niche targeting the sun's range between its nadir, and when it reached the midpoint of the southern skies.

The chimney

There is a third structure which encodes similar solar data. It is to be found in a pyramid built by Khufu's father in Dahshur. The chimney is a subterranean vertical shaft built by King Snefru in his Bent Pyramid. Its dimensions are unusual; its floorplan is a cramped 0.90 m x 1.53 m, however, it rises 15.27 m high⁶³. It contains unusual corbelling and connects to the main chamber via a doorway and a feature called the 'window' (both on the northern side of the chimney).

Its general form is of a rectangular shaft which rises upwards in two distinct stages, in which the form of the bottom half is repeated in reverse in the top half. Corbelling commences on the west and east sides at a height of 393 cm. The shaft juts outwards on both sides at this mark, though this is not symmetrical, as it juts out further on the west side. Then, the corbelling staggers inwards at an even rate on both sides, but since the east side does not jut out as far as the west, it takes just two laps for it to return to being in line with the wall (at the 620 cm mark), however, it requires four laps for the west side to return to being in line with the wall (at the 765 cm mark). The shaft rises upwards from there in its original, rectangular

⁵⁶ J. GREAVES, Miscellaneous Works of Mr. John Greaves: Professor of Astronomy in the University on Oxford I, Oxford, 1737, p. 121.

⁵⁷ W.M.F. PETRIE, *op. cit.*, p. 217.

⁵⁸ M. LEHNER, *The Complete Pyramids*, New York, 1997, p. 113.

⁵⁹ W.M.F. PETRIE, *op. cit.*, p. 188.

⁶⁰ C.P. SMYTH, *op. cit*, p. 185.

⁶¹ S. LABOY, A Civil Engineer looks at the Great Pyramid,

http://www.samuellaboy.com/New_Folder/Special_Topics/Queen%20Chamber's%20design.htm (1 June 2020). ⁶² The error between this observation and 45° is 0.013°.

⁶³ All measures of chimney are taken from V. MARAGIOGLIO, C. RINALDI, L'Architettura Delle Piramidi Menfite III. Il Complesso di Maydum, la Piramide a Doppia Pendenza e las Piramide Settentriola in Pietra di Dahsciur, Torino, 1964, tav. 12.1.

shape, before the corbelled design repeats at 1057 cm. This time, however, the pattern is reversed; the east side juts out further this time, and so requires four laps to return to the straight line, whereas the west requires just two. The chimney terminates at 1527 cm.

Theories on the purpose of the chimney

Various theories have been forwarded regarding the purpose of this structure. According to Dormion and Verd'hurt⁶⁴ the Chimney was originally to have provided a vertical path linking the lower chamber to the upper chamber: "Ce cheminement vertical fut, toutefois, condamné et suggère que l'ensemble du dispositif inférieur a été abandonné au bénéfice de l'appartement supérieur."

According to Monnier and Puchkov⁶⁵ the upper chamber was the intended burial chamber while the lower chamber was just an antechamber. In their opinion:

"the 'chimney' was without doubt initially intended to lead to a burial chamber. ... The architect probably judged that the 'chimney' was too perilous to allow access to the funeral procession, and so an alternative arrangement had to be found."

Another theory concerning the chimney was offered by Moores⁶⁶ who argued that the chimney may have served as a "control datum for guiding the construction of the interior apartments". Fakhry⁶⁷ however, called the chimney "one of the unsolved problems of the pyramid". Its positioning, he argued, certainly suggests that it carried significance, as it is placed at the very centre of the pyramid in Mustapha's survey⁶⁸, as well as in Perring's⁶⁹ and it terminates at ground level. The central point is an important position in a pyramid: it is from this point that the pyramid's perimeter was measured; the chamber in the neighbouring satellite pyramid was placed in the centre; and eventually, when completed, the capstone of the Bent Pyramid was placed 200 cubits directly above this point. Trumpp ⁷⁰ also points to the central positioning of the chimney, arguing that its placement may have carried a solar function "to provide a connection to the pyramid peak and to the Sun God Ra."

⁶⁴ G. DORMION, J.-Y. VERD'HURT, *La Chambre de Snefrou: Analyse Architecturale de la Pyramide Rhomboïdale*, Arles, 2016, p. 7. In their opinion, the dead end at the top of the chimney is indicative of its abandonment.

 $^{^{65}}$ F. MONNIER, A. PUCHKOV, « The Construction Phases of the Bent Pyramid at Dahshur. A Reassessment », *ENiM* 9, 2016, p. 21. The scenario presented by Monnier and Puchkov was that the architect had a sudden realization that funerary access would be too difficult via this chimney and so it was abandoned. The problem with this theory is that it was designed as a small space, rising 15 m upwards, making it difficult to imagine that funerary access was ever a motivation.

⁶⁶ B. MOORES, *Building the Pyramids: How did they do it?*, Bloomington, 2019, n. p.

⁶⁷ A. FAKHRY, The Monuments of Senefru at Dahshur II. The Valley Temple 2. The Finds, Cairo, 1961, p. 102.

⁶⁸ H. MUSTAPHA, "The Surveying of the Bent Pyramid at Dahshur," in A. Fakhry, *The Monuments of Senefru at Dahshur* I. *The Bent Pyramid*, Cairo, 1959, p. 68, fig. 33.

⁶⁹ J.S. PERRING, *The Pyramids of Gizeh* III. *The Pyramids to the Southward of Gizeh and at Abou Roash*, London, 1842, p. 2, pl. xvi.

⁷⁰ H.-J. TRUMPP, *Master Plan of the Pyramid of Cheops: Meaning, Shape and Size Better Understood*, Wisconsin, 2014, p. 24.

Determining the height of the window

According to the hypothesis presented in this paper, the floor and ceiling of the King's Chamber represented the sun's range from solstice to solstice as it crossed the meridian. The chimney appears to have adopted a different method; it may have represented the entire southern skies, from 0° (the horizon) to 90° (directly overhead), and then within that span it used a feature (the window) to target a portion of the sun's range.

The head of the window is 7.65 m up the chimney and it extends downwards from there for 2.20 m, therefore the sill of the window is therefore at a height of 5.45 m above the pavement.⁷¹ However, the window may not be in its original form. Currently, it has a corbelled arrangement spanning three courses of brickwork, with its size reducing with each new course line. Maragioglio and Rinaldi⁷² designate the bottom course (which comprises one single stone block) as "blocco mancanti." Measuring the height of the top two tiers of the window restores its probable original form and reduces its height to 1.45 m.⁷³ Consequently, the base of the window originally began at 6.20 m, a height which Maragioglio and Rinaldi⁷⁴ identify as "soglia originale." The likely scenario leading to the expansion of the window was treasure hunting. The chimney could readily be entered from the chamber as there is a doorway leading from the chamber into it, however, a portcullis stone blocked the chimney at the height of 3.93 m. The window provided an entry into the chimney above this point, however, due to its narrow width (57 cm), the lower stone was removed to enable access.⁷⁵

Determining the solar elevations associated with the head and sill of the window

The chimney is 15.27 m high. If it was intended to represent 90° of sky, then dividing 15.27 m by 90 will yield the number of centimetres per degree: 16.967 cm. Dividing the height of the sill and the head of the window by this number will reveal the solar elevations that they would represent according to the proposed system.

The head of the window is at a height of 7.65 m. $765 \div 16.967 = 45.088^{\circ}$.

The sill of the window is at a height of 6.20 m. $620 \div 16.967 = 36.542^{\circ}$.

When the chimney is regarded as a map of the southern skies, the window demarcates a range of solar elevations from 36.54° to 45.09° .⁷⁶ This is in keeping with the measures preserved in the two chambers in the Great Pyramid. The niche in the Queen's Chamber provided measures of 36.5° and 44.987° ; and the King's Chamber's features provided 36.477° and 45° . These values are very similar and appear to hone in on two specific values: the midpoint in the southern skies (45°); and an estimate of the winter solstice noonday low for the sun in the 26^{th} century BC.

⁷¹ V. MARAGIOGLIO, C. RINALDI, *op. cit.*, *tav.* 12.1.

⁷² *Ibid.*, tav. 12.1.

⁷³ *Ibid.*, tav. 12.1.

⁷⁴ *Ibid.*, tav. 12.1.

⁷⁵ *Ibid.*, tav. 12.1.

 $^{^{76}}$ 145 ÷ 16.92222222 = 8.56861459°. 45° - 8.56861459° = 36.4313855°. Note that the winter solstice elevation for the sun as it crossed the meridian at Dahshur is higher due its lower latitude; the sun had an apparent elevation of 36.25° in the mid-26th century BC (NASA JET PROPULSION LABORATORY, *op. cit.*). This is 0.2° greater than at Gizeh, thus reducing the winter solstice solar elevation error provided by the Chimney from 0.49° to 0.29°.

The importance of the winter skies

The reason why this portion of the skies may have been encoded into these structures was because when the noonday sun was within this span of the skies it was entering the final weeks of its wane, followed by its nadir at the winter solstice, and then the first weeks of its waxing cycle. This period was centred on the winter solstice, a time of the year when many solar worshipping cultures believed that the sun god died and was reborn. Evidence of this belief in Ancient Egypt is attested to by Plutarch⁷⁷ "Isis, when she perceived that she was pregnant, put upon herself an amulet on the sixth day of the month Phaophi; and about the time of the Winter solstice she gave birth to Harpocrates." The importance of the winter solstice is also preserved in Ancient Egyptian architecture; according to Belmonte et al.⁷⁸ aligning temples to the rising sun of the winter solstice was "one of their favourite ways of orientation." Further evidence is presented by Belmonte and Magli⁷⁹ who argued that the view towards the Bent Pyramid from its valley temple created a solar hierophany at sunset on the winter solstice sunset.

Conclusion

This paper presents the hypothesis that the section of the skies between 36.5° and 45° was of particular importance to the Egyptians of Dynasty 4. While there is no written documentary evidence attesting to the integration of solar data into their pyramid designs, architectural parallels found in various chambers within the Great Pyramid and the Bent Pyramid has shown that it is entirely possible that this aspiration dictated the vertical arrangements within these shafts. The features in the King's Chamber, the window, and the niche, all terminate at heights commensurate with 45°, while the southern shaft of the King's Chamber is inclined at this angle. In turn, the winter solstice noonday solar elevation of 36.5° appears to be represented by the floors of both the Queen's Chamber and the King's Chamber, as well as by the base of the window (in its original form). The mathematical accuracy of the observations derived from the Queen's Chamber, the King's Chamber and the Chimney makes for a compelling case that these structures were designed to preserve specific meridional solar elevations in their vertical arrangement.

⁷⁷ PLUTARCH, *Moralia* (F. Babbitt trans.), Cambridge MA, 1936, V.65. Harpocrates was the youthful version of the sun god 'Horus'.

⁷⁸ J. BELMONTE, M. FEKRI, Y. ABDEL-HADI, M. SHALTOUT, A.C.G. GARCIA, "On the Orientation of Ancient Egyptian Temples 5: Testing the Theory in Middle Egypt and Sudan," *JHA* 41/1, 2010, p. 70.

⁷⁹ J. BELMONTE, G. MAGLI, "Astronomy, Architecture, and Symbolism: The Global Project of Sneferu at Dahshur," *JHA* 46/2, 2015, fig. 2f.